Turbulence modeling for wind turbine wakes in non-neutral and anisotropic conditions

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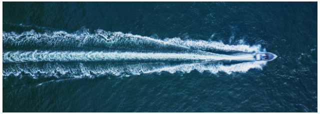
Wakes

- Wakes occur many places in nature

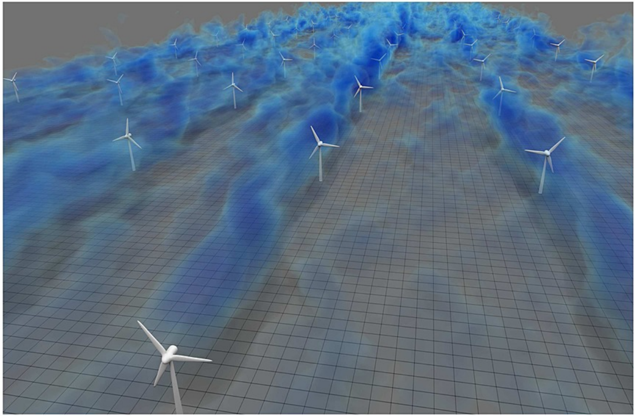


- Wind turbine wakes are typically invisible to the human eye!

Visualization of wind turbine wakes in a wind farm

Wake effects:

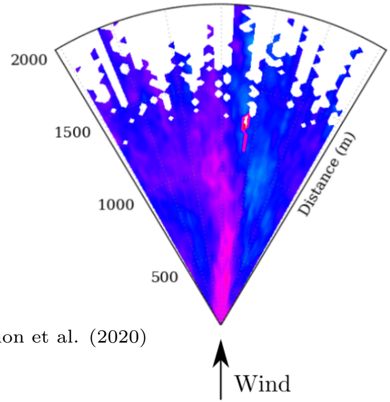
- Low velocity → decreased wind farm power production
- Turbulent motion → shorter turbine lifetime



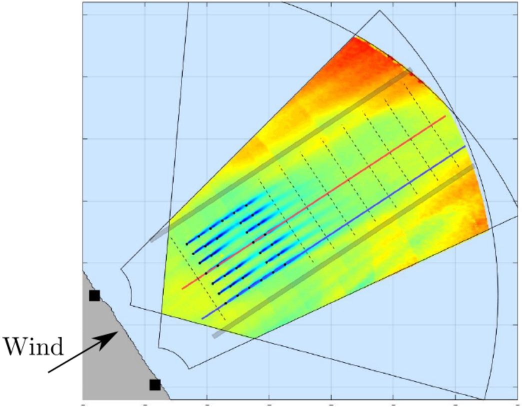
Stevens et. al (2014)

Measurement of wind turbine wakes

- Some experimental evidence of wind turbine wakes



Simon et al. (2020)

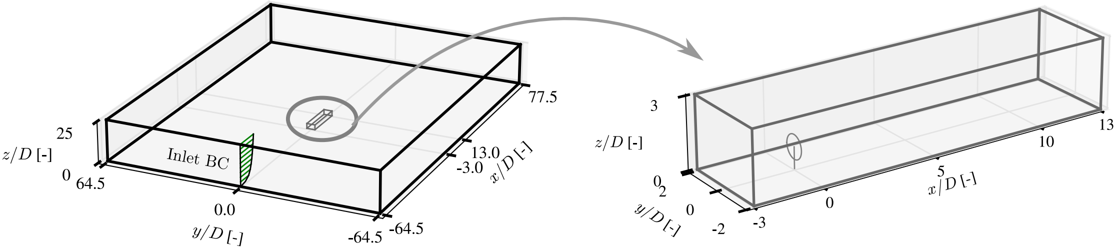


Nygaard and Newcombe (2018)

Simulation of wind farm flow

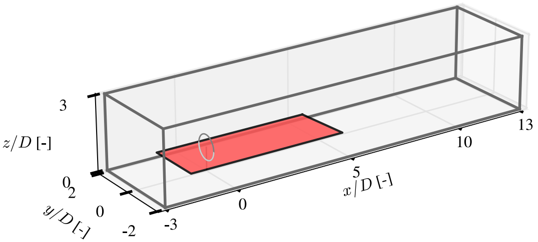
Several choices of:

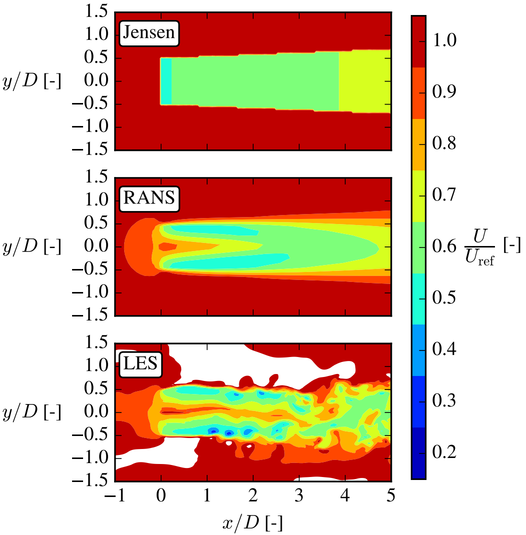
1. Simulation method
2. Atmospheric profiles set at the inlet



Different simulation methods

- Trade-off between flow details and computational cost

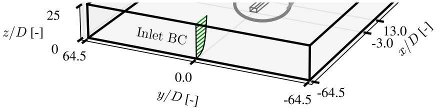


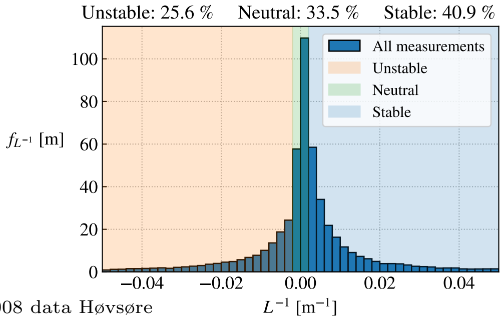


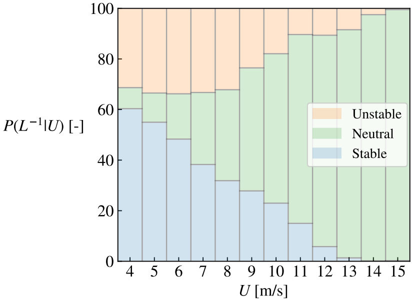
Atmospheric profiles depend on the state of the atmosphere

There are roughly three different states:

- Neutral: No buoyancy effects
- Unstable: Turbulence added by buoyancy
- Stable: Turbulence dampened by buoyancy



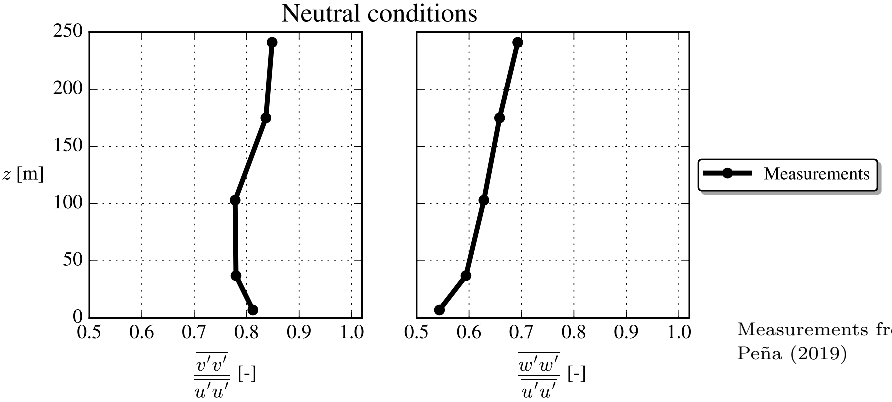




2008 data Høvsøre

Anisotropic turbulence in the atmosphere

- There tends to be more turbulence in some directions ("anisotropy") in the atmosphere: $\overline{u'u'} > \overline{v'v'} > \overline{w'w'}$



Measurements from Peña (2019)

- This phenomenon is not possible to model with standard RANS models

Research objectives

Title of my PhD:

$$\text{“}\underbrace{\text{Turbulence modeling}}_{\text{RANS}} \text{ for } \underbrace{\text{wind turbine wakes}}_{\text{important for wind farms}} \text{ in } \underbrace{\text{non-neutral and anisotropic conditions}}_{\text{more realistic atmospheric conditions}}\text{”}$$

- Task 1: RANS simulation of wakes in non-neutral conditions
  - Revise the k-ε-f_P MOST model by van der Laan et al. (2017)
- Task 2: RANS simulation of wakes in anisotropic conditions
  - Need a more advanced turbulence model → will use the explicit algebraic Reynolds stress model (EARSM) by Wallin & Johansson (2000)
  - Will only consider neutral conditions

RANS - simulating the mean flow directly!

- RANS idea: Time-average the governing equations first, before solving anything

Reynolds decomposition:

$$\tilde{u}_i = U_i + u_i'$$

Navier-Stokes

$$\frac{\partial \tilde{u}_i}{\partial x_i} = 0$$
$$\frac{D\tilde{u}_i}{Dt} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial x_i} + \frac{\partial}{\partial x_j}(2\nu \tilde{s}_{ij})$$

$\xrightarrow{\text{time avg}}$

Reynolds-Averaged Navier-Stokes (RANS)

$$\frac{\partial U_i}{\partial x_i} = 0$$
$$\frac{DU_i}{Dt} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}(2\nu S_{ij} - \boxed{\overline{u_i' u_j'}})$$

- To simulate with RANS, we need *turbulence modeling* for the last term

An example of a turbulence model: The k-ε-f_P model

- Based on the classic k-ε model and adapted to wind farm flows by van der Laan (2014)
- Only valid for neutral conditions!

Step 1: Turbulent transport equations

$$\frac{Dk}{Dt} = \mathcal{P} - \varepsilon + \mathcal{D}^{(k)}$$
$$\frac{D\varepsilon}{Dt} = (C_{\varepsilon 1}\mathcal{P} - C_{\varepsilon 2}\varepsilon)\frac{\varepsilon}{k} + \mathcal{D}^{(\varepsilon)}$$

Step 2: Eddy viscosity

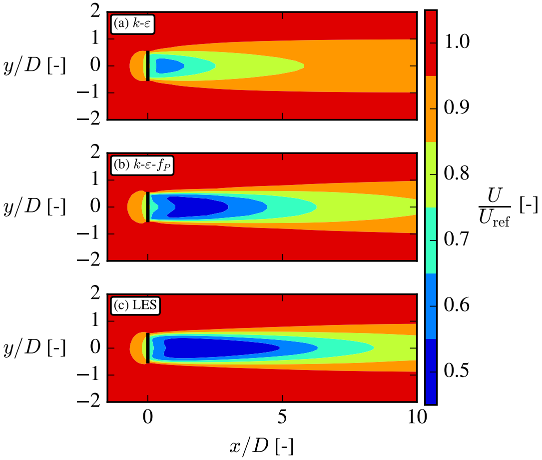
$$f_P = f\left(k, \varepsilon, \frac{\partial U_i}{\partial x_j}\right)$$
$$\nu_t = C_\mu f_P \frac{k^2}{\varepsilon}$$

Step 3: Boussinesq hypothesis

$$\overline{u_i' u_j'} = -\nu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) + \frac{2}{3}k\delta_{ij}$$

The k-ε-f_P model in action for a neutral case

- Simulation of a V80 turbine with EllipSys3D (RANS) and compared to LES (Aarhus University code)



Realizability

- A turbulence model can sometimes give unphysical turbulence

Realizable turbulence

$$0 \leq \overline{u'_\alpha u'_\alpha} \leq 2k$$

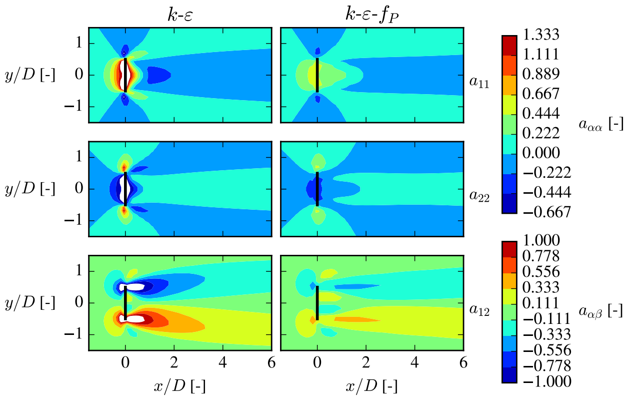
$$\left(\overline{u'_\alpha u'_\beta}\right)^2 \leq \overline{u'_\alpha u'_\alpha}\;\overline{u'_\beta u'_\beta} \leq k^2$$

or equivalently

$$-\frac{2}{3} \leq a_{\alpha\alpha} \leq \frac{4}{3}$$

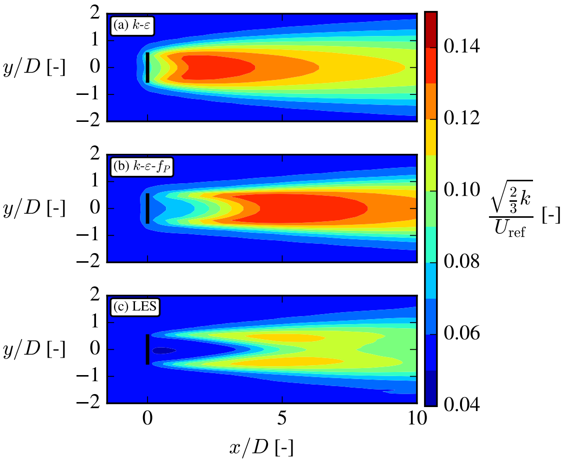
$$-1 \leq a_{\alpha\beta} \leq 1$$

$$a_{ij} \equiv \frac{\overline{u'_i u'_j}}{k} - \frac{2}{3}\delta_{ij}$$



Overprediction of turbulence intensity

- Both k-ε and k-ε-f_P models tend to overpredict turbulence intensity (TI)

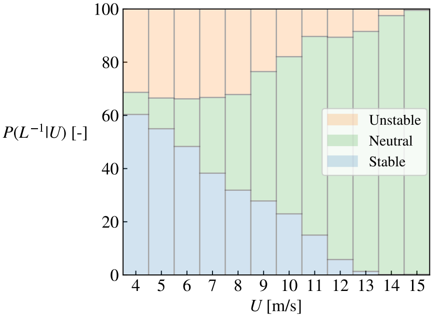


Extending the neutral setup to non-neutral conditions

- Task 1: RANS simulation of wakes in non-neutral conditions

Buoyant production of turbulence

- Unstable: $\mathcal{B} > 0$
- Neutral: $\mathcal{B} = 0$
- Stable: $\mathcal{B} < 0$



- k-ε-f_P MOST (van der Laan et al, 2017) is combination of:
  - k-ε-f_P model (van der Laan, 2014)
  - Monin-Obukhov similarity theory (MOST) (1954)

$\xrightarrow{\text{modify}}$

1. Turbulence model
2. Inflow model

The k-ε-f_P MOST turbulence model

Modified Step 1:

$$\frac{Dk}{Dt} = \mathcal{P} - \varepsilon + \mathcal{D}^{(k)} + \boxed{\mathcal{B}} - \boxed{S_k}$$
$$\frac{D\varepsilon}{Dt} = (C_{\varepsilon 1}\mathcal{P} - C_{\varepsilon 2}\varepsilon + \boxed{C_{\varepsilon 3}}\ \boxed{\mathcal{B}})\frac{\varepsilon}{k} + \mathcal{D}^{(\varepsilon)}$$

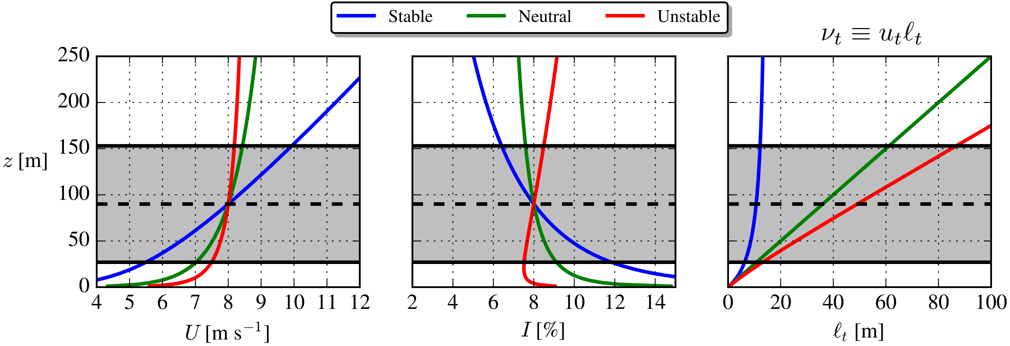
- $\mathcal{B}$ is the buoyant production or destruction of TKE ("indirect" buoyant forcing)

$$\mathcal{B} = -\nu_t \left(\frac{\partial U}{\partial z}\right)^2 \frac{\zeta \Phi_h}{\sigma_\theta \Phi_m^2}$$

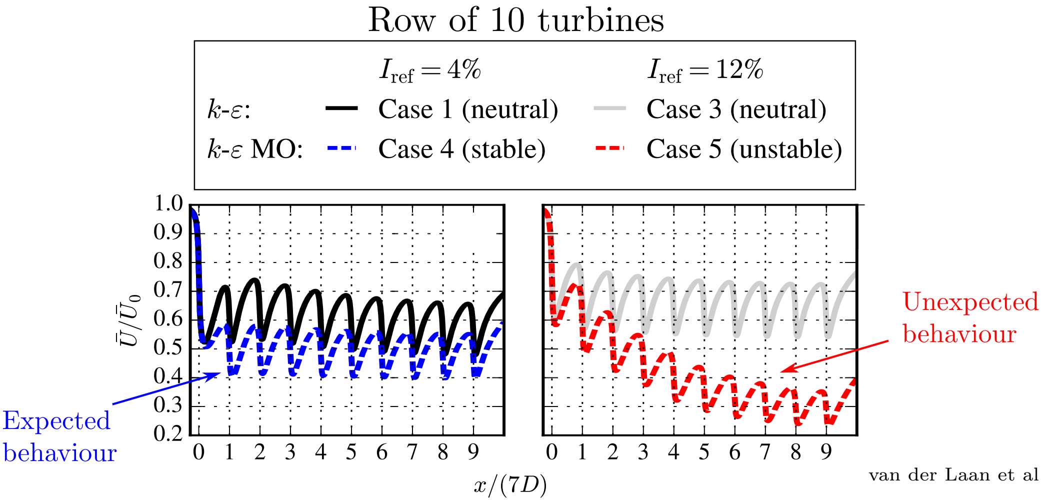
- The "direct" buoyant forcing term in the vertical momentum equation is neglected in this model

Non-neutral inflow with MOST

- Modified inflow profiles using MOST



Application of the k-ε-f_P MOST model to a row of turbines



van der Laan et al. (2021)

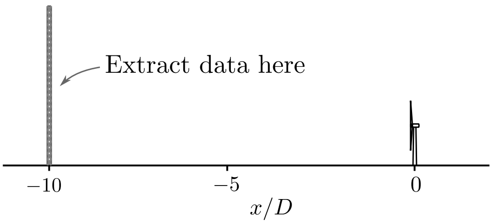
Investigating the unstable k-ε-f_P MOST model (Paper 1)

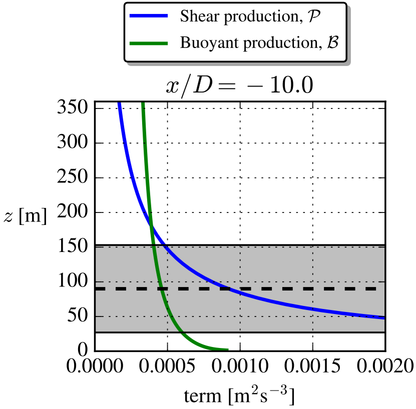
Problem 1:

- Freestream: $\mathcal{B}$ changes with height

→ Unexpected

  - MOST should have $\mathcal{B} = -\frac{u_*^3}{\kappa L}$





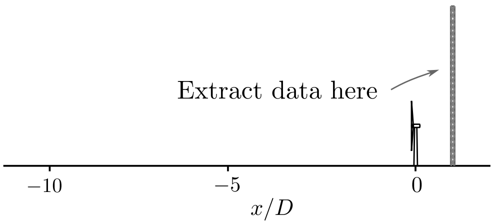
Investigating the unstable k-ε-f_P MOST model

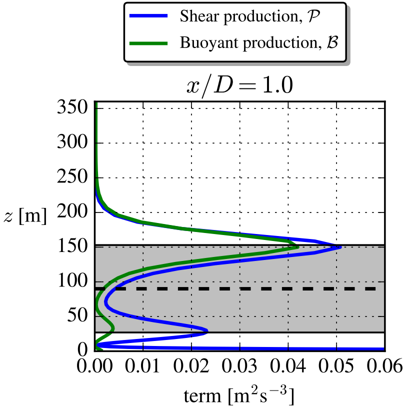
Problem 2:

- Near-wake: $\mathcal{B}$ seems to scale with $\mathcal{P}$

$\rightarrow$ Unexpected

  - Wind tunnel experiments show that $\mathcal{B} \ll \mathcal{P}$ in the near-wake (Hancock and Zhang, 2015)
  - LES also show that $\mathcal{B}/\mathcal{P} = \mathcal{O}(0.01)$ in the wake shear layers!



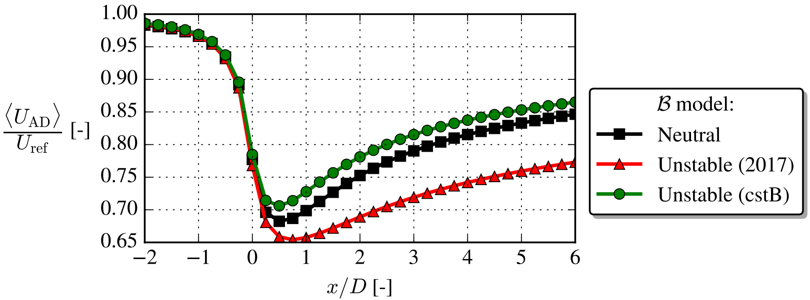


The constant $\mathcal{B}$ model

- A simple way to fix the two problems:

$$\cancel{\mathcal{B} = -\nu_t \left(\frac{\partial U}{\partial z}\right)^2 \frac{\zeta \Phi_h}{\sigma_\theta \Phi_m^2}} \quad , \quad \mathcal{B} = -\frac{u_*^3}{\kappa L}$$

- Exact in the freestream
- Only a first order approximation in the wake, but $\mathcal{P}$ dominates there anyway

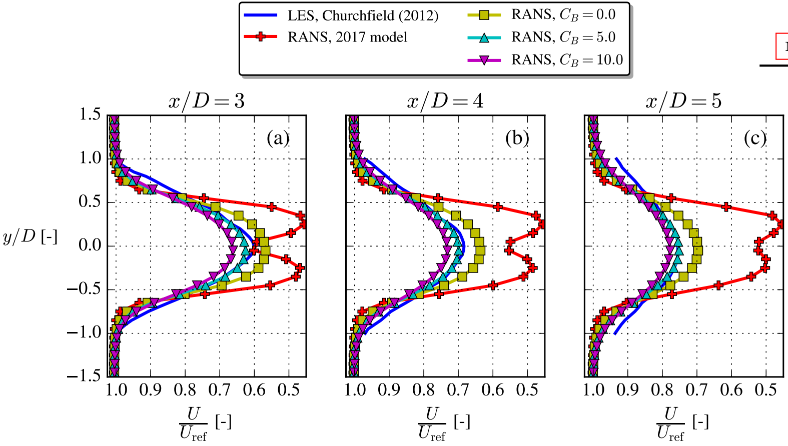


Validation of the k-ε-f_P MOST "cstB" model

- Tested the new model on five cases from the literature
- Added a constant, C_B, for fine-tuning

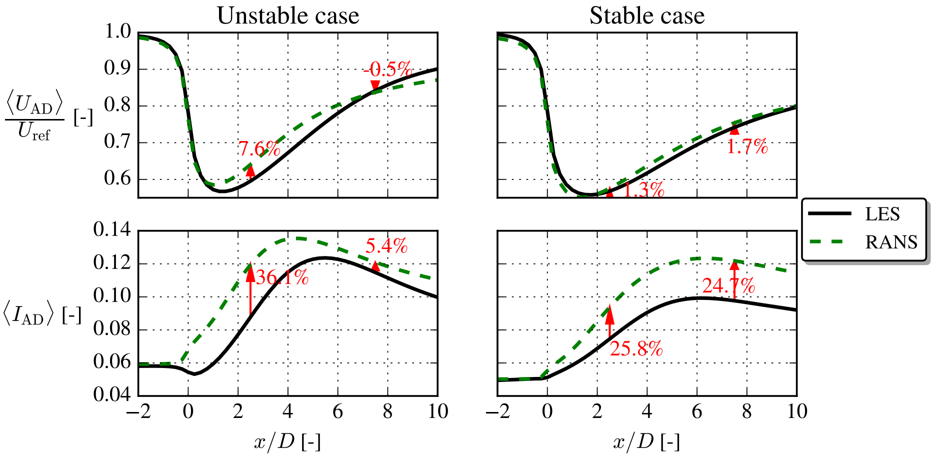
Validation cases

Case	Type
SWiFT	LES, Exp.
NTK41	LES, Exp.
V80-Abkar	LES
V80-Keck	LES
NREL5MW	LES



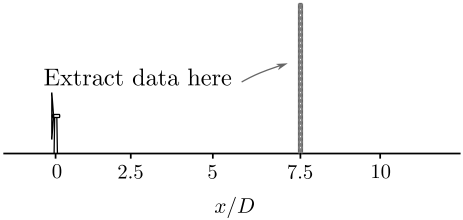
More testing of the model (Paper 2)

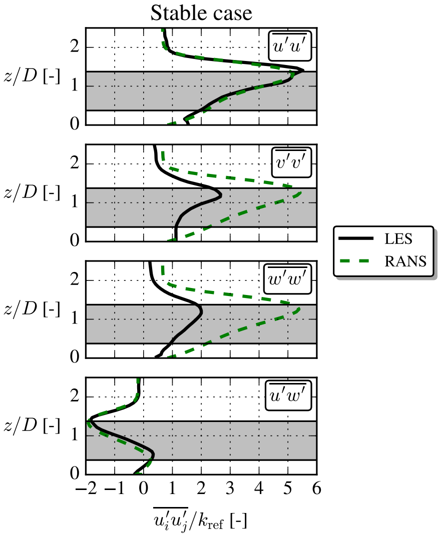
- Did a more detailed comparison study with new LES runs



Reynolds stresses

- Normal stresses overestimated
  $\rightarrow k = \frac{1}{2}\overline{u_i'u_i'}$ and TI overestimated
- Shear stresses compares better
  $\rightarrow$ Velocity deficit compares better

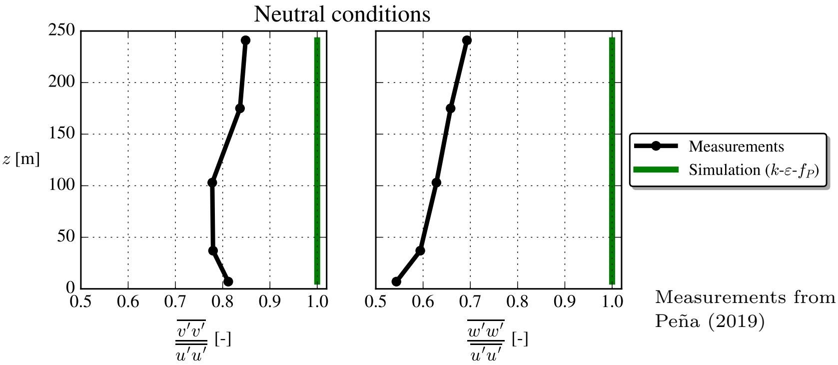




Anisotropic conditions

- Task 2: RANS simulation of wakes in anisotropic conditions

$$\overline{u'u'} > \overline{v'v'} > \overline{w'w'}$$



Measurements from Peña (2019)

- The k-ε-f_P model simply predicts $\overline{u'u'} = \overline{v'v'} = \overline{w'w'}$

Anisotropy-limitation of the Boussinesq hypothesis

Step 3: Boussinesq hypothesis

$$\overline{u_i'u_j'} = -\nu_t \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) + \frac{2}{3} k \delta_{ij}$$

- The normal components in the freestream (horizontally homogeneous flat terrain):

$$\begin{aligned}\overline{u_\alpha' u_\alpha'} &= -\nu_t \left(2 \cancelto{0}{\frac{\partial U_\alpha}{\partial x_\alpha}} \right) + \frac{2}{3} k \\ &= \frac{2}{3} k\end{aligned}$$

- No matter the model for ν_t, the TKE is always split equally between the three components in the freestream!

A more general constitutive relation

- Pope (1975) proved that there is a more general, but finite expression for $\overline{u_i' u_j'}$ (or equivalently for $a_{ij} \equiv \frac{\overline{u_i' u_j'}}{k} - \frac{2}{3}\delta_{ij}$):

$$\boxed{a_{ij} = \sum_{l=1}^{10} \beta_l T_{ij}^{(l)}}$$

Tensor basis
$\mathbf{T}^{(1)} = \mathbf{S}$
$\mathbf{T}^{(2)} = \mathbf{S}^2 - \frac{1}{3} II_S \mathbf{I}$
$\mathbf{T}^{(3)} = \mathbf{\Omega}^2 - \frac{1}{3} II_\Omega \mathbf{I}$
$\mathbf{T}^{(4)} = \mathbf{S}\mathbf{\Omega} - \mathbf{\Omega}\mathbf{S}$
$\mathbf{T}^{(5)} = \mathbf{S}^2\mathbf{\Omega} - \mathbf{\Omega}\mathbf{S}^2$
$\mathbf{T}^{(6)} = \mathbf{S}\mathbf{\Omega}^2 + \mathbf{\Omega}^2\mathbf{S} - \frac{2}{3} IV \mathbf{I}$
$\mathbf{T}^{(7)} = \mathbf{S}^2\mathbf{\Omega}^2 + \mathbf{\Omega}^2\mathbf{S}^2 - \frac{2}{3} V \mathbf{I}$
$\mathbf{T}^{(8)} = \mathbf{S}\mathbf{\Omega}\mathbf{S}^2 - \mathbf{S}^2\mathbf{\Omega}\mathbf{S}$
$\mathbf{T}^{(9)} = \mathbf{\Omega}\mathbf{S}\mathbf{\Omega}^2 - \mathbf{\Omega}^2\mathbf{S}\mathbf{\Omega}$
$\mathbf{T}^{(10)} = \mathbf{\Omega}\mathbf{S}^2\mathbf{\Omega}^2 - \mathbf{\Omega}^2\mathbf{S}^2\mathbf{\Omega}$

- What should the coefficients, β_l, be?
  - Set $\beta_{\{2-10\}} = 0 \rightarrow$ “Linear eddy-viscosity model (EVM)”
  - Tune with data $\rightarrow$ “Non-linear EVM (NLEVM)”
  - Obtain from simplification of differential Reynolds stress model (DRSM) $\rightarrow$ “EARSM”

Derivation of an explicit algebraic Reynolds stress model (EARSM)

$$\frac{Da_{ij}}{Dt} = \dots$$

Differential Reynolds stress models (DRSM)

↓

$$a_{ij} = f_1(a_{ij}, S_{ij}, \Omega_{ij})$$

Algebraic RSM (ARSM)

↓

$$a_{ij} = f_2(S_{ij}, \Omega_{ij})$$

Explicit algebraic RSM (EARSM)

- Independent breakthroughs by Wallin & Johansson (1996), Girimaji (1996) and Ying & Canuto (1996) regarding the non-linearity

Comparing the WJ-EARSM with some linear EVMs

- I focused on the EARSM by Wallin & Johansson (2000), which can be written as:

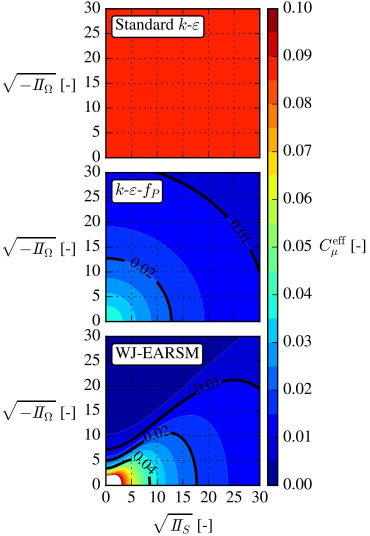
$$a_{ij} = -2C_\mu^{\mathrm{eff}} S_{ij} + a_{ij}^{(\mathrm{ex})}$$

Model	C_μ^{eff}	$a_{ij}^{(\mathrm{ex})}$
k-ε	C_μ	0
k-ε-f_P	$C_\mu f_P(\mathit{II}_S, \mathit{II}_\Omega)$	0
2D WJ-EARSM	$f(\mathit{II}_S, \mathit{II}_\Omega)$	$g_1(\beta_l, T_{ij}^{(l)})$
3D WJ-EARSM	$f(\mathit{II}_S, \mathit{II}_\Omega)$	$g_2(\beta_l, T_{ij}^{(l)})$

$$\mathit{II}_S \equiv S_{ij}S_{ji} \quad , \quad \mathit{II}_\Omega \equiv \Omega_{ij}\Omega_{ji}$$

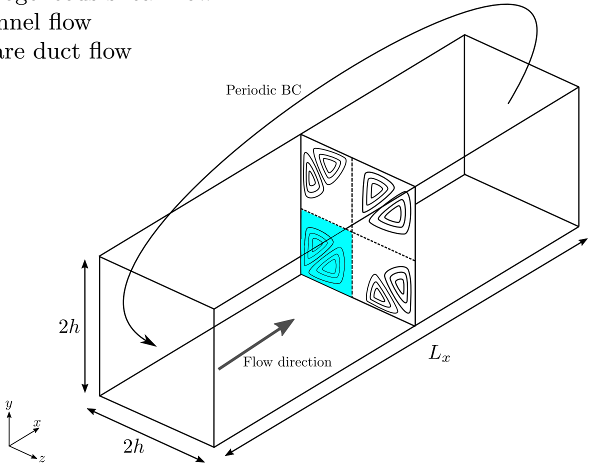
$$-\frac{2}{3} \leq a_{\alpha\alpha} \leq \frac{4}{3}$$

$$-1 \leq a_{\alpha\beta} \leq 1$$



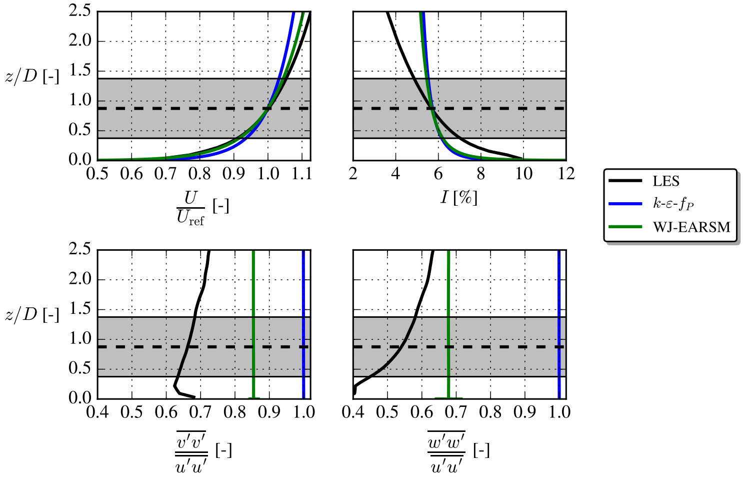
Model verification (Paper 3)

- Used three basic flows to verify the code implementation
  - Homogeneous shear flow
  - Channel flow
  - Square duct flow



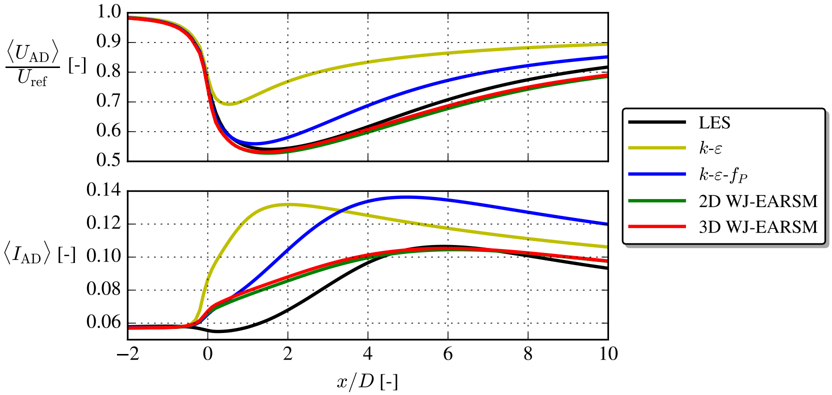
Comparison of neutral inflow profiles

- WJ-EARSM is able to predict freestream anisotropy!



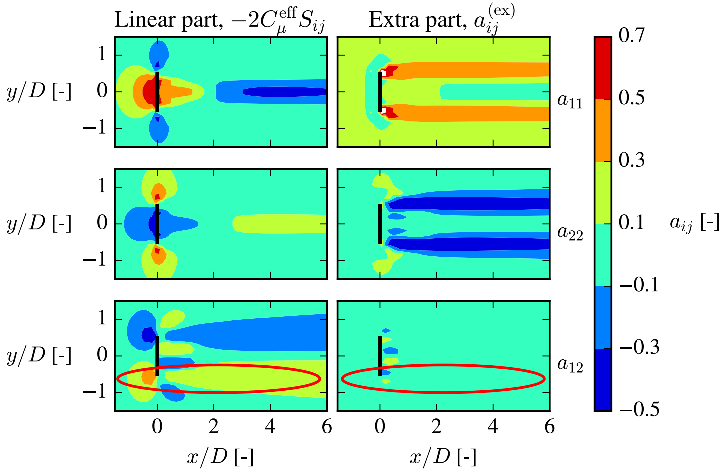
Disk-averaged recovery

- Better velocity deficit and TI predictions with WJ-EARSM
- The 2D WJ-EARSM gives almost the same results as the more complicated 3D WJ-EARSM



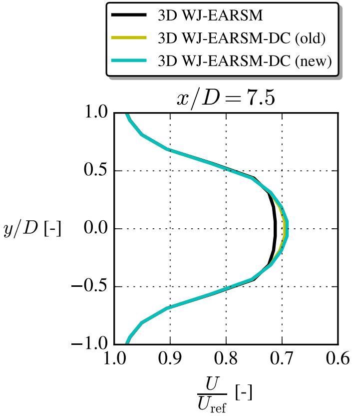
Anisotropy split

- A possible explanation of the similar behavior of the 2D and 3D WJ-EARSMs



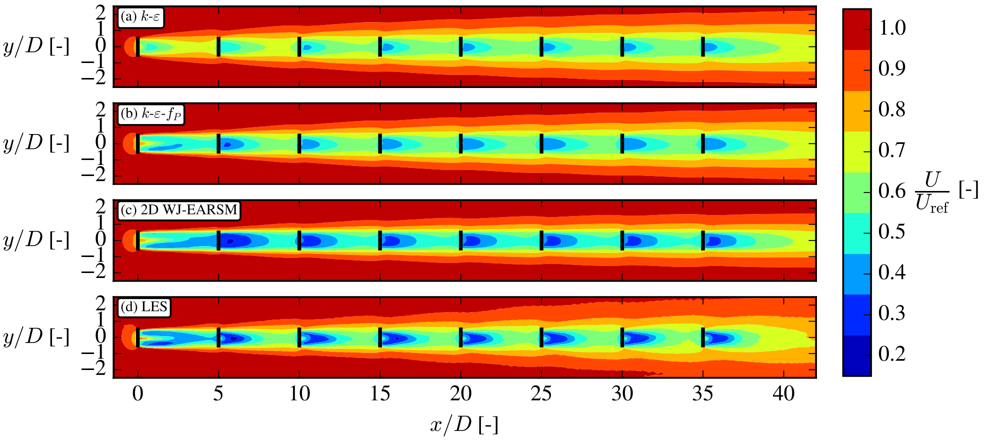
Tendency of “top-hat” shaped velocity profile

- A flattened wake center was observed in the WJ-EARSM simulations, which can be corrected in different ways:
  - Tuning the Rotta coefficient
  - Taking wind direction uncertainty into account
  - Diffusion correctio

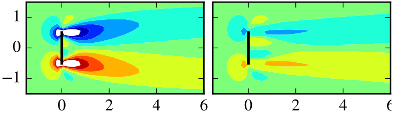


Application of WJ-EARSM to a row of turbines

- The WJ-EARSM is numerically stable also for larger cases



Conclusions



- The success of the k-ε-f_P model is connected with its realizability property
- Task 1: The k-ε-f_P MOST model has been revised to simulate wind turbine wakes in non-neutral conditions
  - Based on the observation that $\mathcal{B} \ll \mathcal{P}$ in the wake shear layer
  - Improved wake velocity deficit prediction for a range of validation cases
- Task 2: The WJ-EARS model (2000) has been utilized to simulate wind turbine wakes in anisotropic conditions
  - More complete description of the Reynolds stresses at the same cost as traditional two-equation models
  - Promising results for neutral conditions

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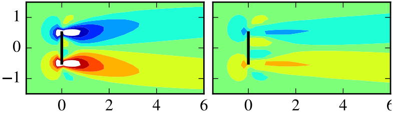

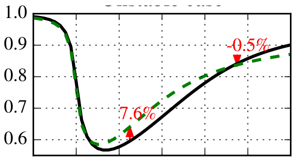

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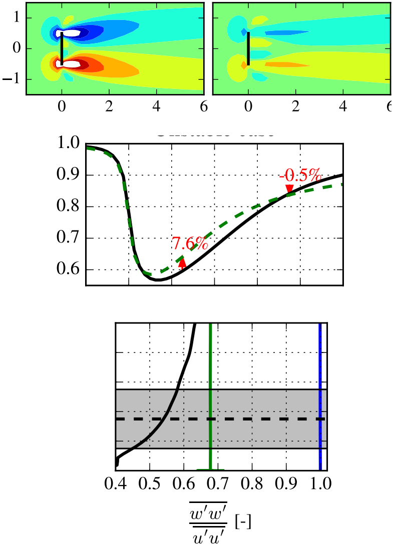
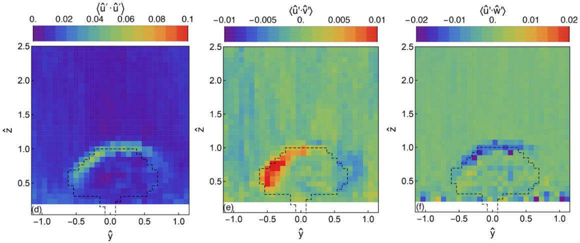
  - Improved wake velocity deficit prediction for a range of validation cases
- Task 2: The WJ-EARS model (2000) has been utilized to simulate wind turbine wakes in anisotropic conditions
  - More complete description of the Reynolds stresses at the same cost as traditional two-equation models
  - Promising results for neutral conditions



The end for now..

Some outlooks:

- Using pressure-driven boundary layer for RANS simulations
- URANS with the non-neutral extension of the WJ-EARSM by Lazeroms (2015) and Zeli (2021)
- Comparison with advanced remote sensing measurements



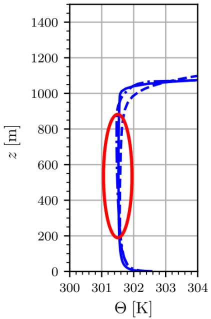
Angelou et al. (2021)

Boundary-Layer Meteorology (2021) 178:487–497
https://doi.org/10.1007/s10546-020-00580-3

NOTES AND COMMENTS

Explicit Algebraic Reynolds-stress Modelling of a Convective Atmospheric Boundary Layer Including Counter-Gradient Fluxes

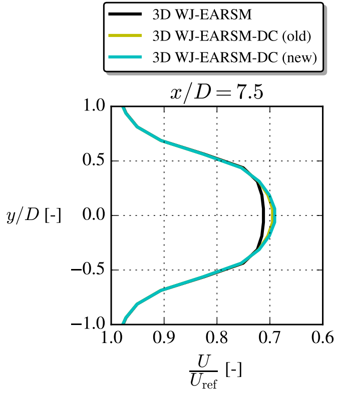
Velibor Želi[1] · Geert Brethouwer[1] · Stefan Wallin[1] · Arne V. Johansson[1]



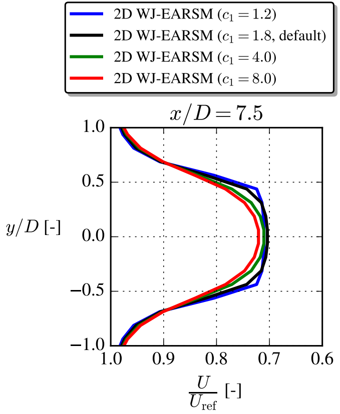
Tendency of "top-hat" shaped velocity profile (expanded)

- A flattened wake center was observed in the WJ-EARSM simulations

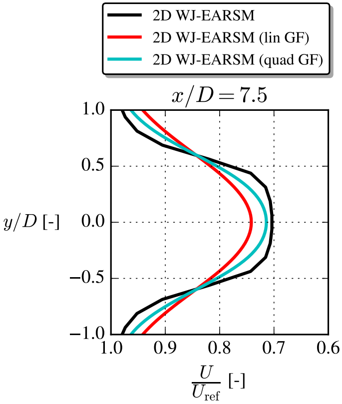
Diffusion correction



The Rotta coefficient



Wind direction uncertainty



LES setup

- The LES code is a version of the code from the Porté-Agel group
- Spectral discretization in horizontal directions, FD in vertical direction
- Fringe region technique used to introduce precursor flow
- Domain size, $L_x/D = 60$, $L_y/D = 10$ and $L_z/D = 5$
- Uniform spatial resolution, $\Delta_x/D = 8$, $\Delta_y/D = \Delta_z/D = 16$
- Periodic BCs in horizontal, symmetry top BC and rough wall BC.
- Adams-Bashforth time integration
- Conservative time step throughout domain, $\frac{U\Delta t}{\Delta_x} = 0.06$
- LASD SGS model
- Averaging time is 20 flowthrough times, $\frac{\Delta t_{\mathrm{ave}}}{L_x/U_{\mathrm{ref}}} = 20$
- Turbine modeled as AD with uniform loading and using 1D mom'm controller

RANS setup

- EllipSys3D FV code
- SIMPLE method with modified Rhie-Chow algorithm
- Domain size, $L_x/D = 142$, $L_y/D = 129$ and $L_z/D = 25$
- Wake domain size, $l_x/D = 16$, $l_y/D = 3$ and $l_z/D = 3$
- Wake domain spatial resolution, $\Delta_x/D = \Delta_y/D = 10$
- Grid is stretched in vertical direction and outwards from wake domain using hyperbolic tangent method (Thompson, 1985)
- Inlet BC, outlet BC, periodic side BCs, inlet top BC and rough wall BC.
- Turbine modeled as AD with uniform loading and fixed force control