Modelling of conventionally neutral boundary layers with an explicit algebraic Reynolds stress model

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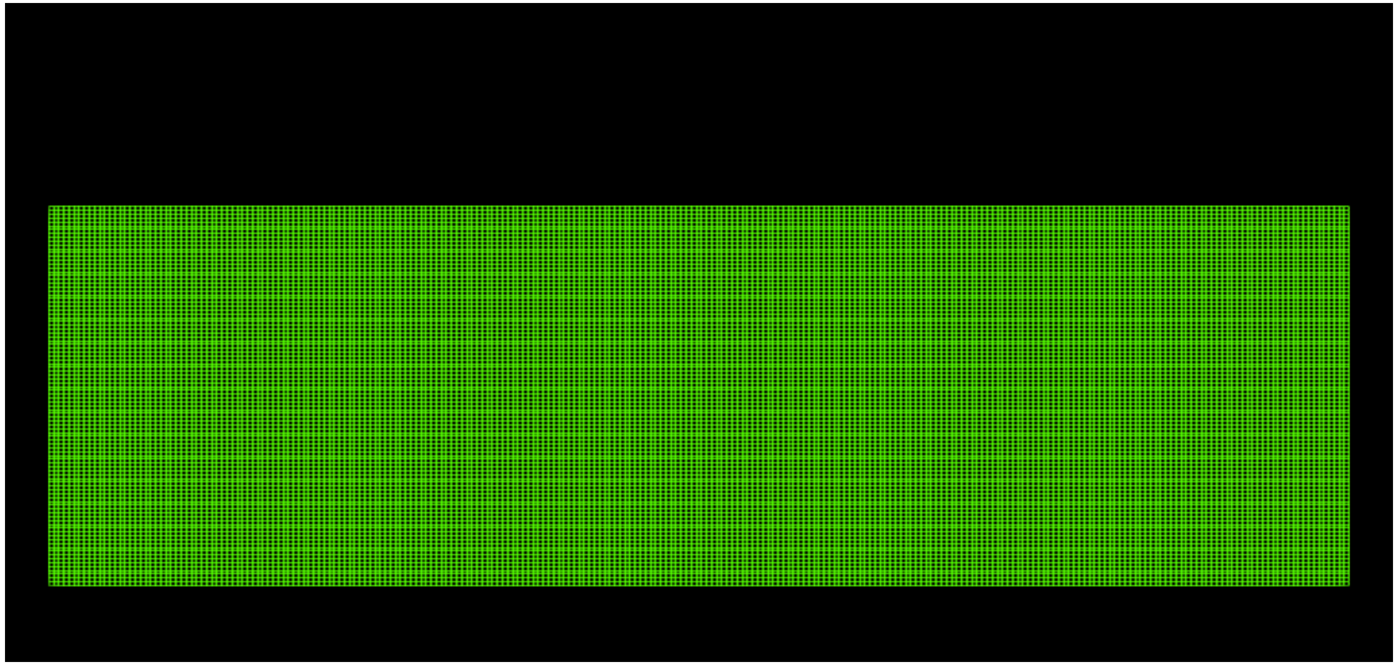
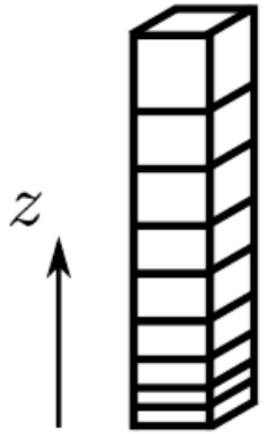
20 August 2025

[1]Baungaard, M, M P Van Der Laan, M Kelly, and E L Hodgson. "Simulation of a Conventionally Neutral Boundary Layer with Two-Equation URANS." *Journal of Physics: Conference Series* 2767, no. 5 (2024): 052013. https://doi.org/10.1088/1742-6596/2767/5/052013.

Motivation

LES is nice, but expensive.





LES (EllipSys3D):
4500 core-hours

Cost factor[1]: $\mathcal{O}(10^6)$

1D URANS (EllipSys1D):
25 core-seconds

Single-column equations

For <u>horizontally homogeneous</u> flows.

https://grevilleawindfarm.com.au/

RANS is cheap, but sensitive to turbulence model.

$$\frac{\partial U}{\partial t} = f(V - V_g) - \frac{\partial \overline{uw}}{\partial z},$$

$$\frac{\partial V}{\partial t} = f(U_g - U) - \frac{\partial \overline{vw}}{\partial z},$$

$$\frac{\partial \Theta}{\partial t} = - \frac{\partial \overline{w\theta}}{\partial z},$$

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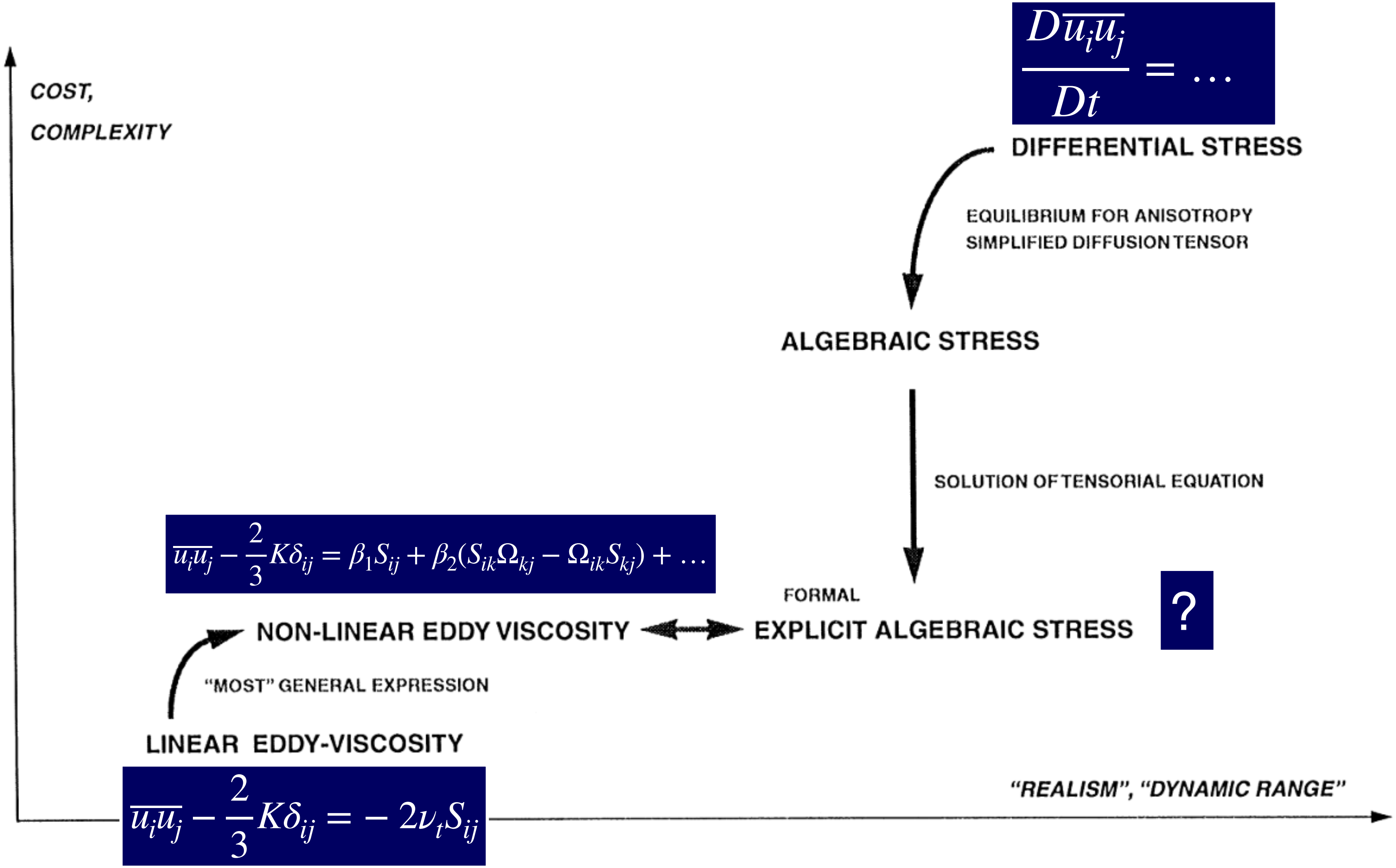
Gatski, T. B., and T. Jongen. “Nonlinear Eddy Viscosity and Algebraic Stress Models for Solving Complex Turbulent Flows.” *Progress in Aerospace Sciences* 36, no. 8 (2000): 655–82. https://doi.org/10.1016/S0376-0421(00)00012-9.

Turbulence model classes



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EARSM at KTH

2000

Neutral EARSM

Wallin, Stefan, and Arne V. Johansson. "An Explicit Algebraic Reynolds Stress Model for Incompressible and Compressible Turbulent Flows." *Journal of Fluid Mechanics* 403 (2000): 89–132. https://doi.org/10.1017/S0022112099007004.

Passive scalar EARSM

Wikström, P. M., S. Wallin, and A. V. Johansson. "Derivation and Investigation of a New Explicit Algebraic Model for the Passive Scalar Flux." *Physics of Fluids* 12, no. 3 (2000): 688–702. https://doi.org/10.1063/1.870274.

2013-2016

Active scalar EARSM

Lazeroms, W. M.J., G. Brethouwer, S. Wallin, and A. V. Johansson. "An Explicit Algebraic Reynolds-Stress and Scalar-Flux Model for Stably Stratified Flows." *Journal of Fluid Mechanics* 723 (2013): 91–125. https://doi.org/10.1017/jfm.2013.116.

Lazeroms, Werner M J. "Turbulence Modelling Applied to the Atmospheric Boundary Layer." KTH Royal Institute of Technology, 2015.

Lazeroms, W. M.J., G. Svensson, E. Bazile, G. Brethouwer, S. Wallin, and A. V. Johansson. "Study of Transitions in the Atmospheric Boundary Layer Using Explicit Algebraic Turbulence Models." *Boundary-Layer Meteorology* 161, no. 1 (2016): 19–47. https://doi.org/10.1007/s10546-016-0194-1.

2019-2021

Test for different ABLs

Želi, Velibor, Geert Brethouwer, Stefan Wallin, and Arne V. Johansson. "Consistent Boundary-Condition Treatment for Computation of the Atmospheric Boundary Layer Using the Explicit Algebraic Reynolds-Stress Model." *Boundary-Layer Meteorology* 171, no. 1 (April 15, 2019): 53–77. https://doi.org/10.1007/s10546-018-0415-x.

Želi, Velibor, Geert Brethouwer, Stefan Wallin, and Arne V. Johansson. "Modelling of Stably Stratified Atmospheric Boundary Layers with Varying Stratifications." *Boundary-Layer Meteorology* 176, no. 2 (August 1, 2020): 229–49. https://doi.org/10.1007/s10546-020-00527-8.

Želi, Velibor. "Modelling of Stably-Stratified, Convective and Transitional Atmospheric Boundary Layers Using the Explicit Algebraic Reynolds-Stress Model." KTH Royal Institute of Technology, 2021.

EARSM at KTH

2021-2023

Implementation of neutral EARSM in EllipSys

Baungaard, Mads, Stefan Wallin, Maarten Paul van der Laan, and Mark Kelly. "Wind Turbine Wake Simulation with Explicit Algebraic Reynolds Stress Modeling." *Wind Energy Science* 7, no. 5 (October 10, 2022): 1975–2002. https://doi.org/10.5194/WES-7-1975-2022.

Baungaard, M., M. P. van der Laan, S. Wallin, and M. Abkar. "RANS Simulation of a Wind Turbine Wake in the Neutral Atmospheric Pressure-Driven Boundary Layer." *Journal of Physics: Conference Series* 2505, no. 1 (2023). https://doi.org/10.1088/1742-6596/2505/1/012028.

2024-2025

Buoyant EARSM in Python 1d-solver

- Ekman and Ellison flow
- Channel flow with active scalar
- GABLS1
- CNBL
- …

What is EARSM?

There are two elements:

1) *Scale-determining equations* $\rightarrow K, \tau$ *and* K_θ

2) *Constitutive relations* $\rightarrow \overline{u_i u_j}$ *and* $\overline{u_i \theta}$

Scale-determining equations

We use a $K - \varepsilon - K_\theta$ model:

$$\frac{\partial K}{\partial t} = \underbrace{-\overline{uw}\frac{\partial U}{\partial z} - \overline{vw}\frac{\partial V}{\partial z}}_{\mathscr{P}} - \varepsilon + \underbrace{\frac{g}{T_0}\overline{w\theta}}_{\mathscr{G}} + \underbrace{\frac{\partial}{\partial z}\left(\frac{\nu_t}{\sigma_K}\frac{\partial K}{\partial z}\right)}_{\mathscr{D}_K}$$

$$\frac{\partial \varepsilon}{\partial t} = C_{\varepsilon 1}\frac{\varepsilon}{K}\mathscr{P} - C_{\varepsilon 2}\frac{\varepsilon^2}{K} + C_{\varepsilon 3}\frac{\varepsilon}{K}\mathscr{G} + \underbrace{\frac{\partial}{\partial z}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial z}\right)}_{\mathscr{D}_\varepsilon}$$

Turbulent scales:

K

$\tau = \dfrac{K}{\varepsilon}$

K_θ

$$\frac{\partial K_\theta}{\partial t} = \underbrace{\overline{w\theta}\frac{\partial \Theta}{\partial z}}_{\mathscr{P}_\theta} - \underbrace{\frac{K_\theta}{rK}\varepsilon}_{\varepsilon_\theta} + \underbrace{\frac{\partial}{\partial z}\left(\frac{\nu_t}{\sigma_{K_\theta}}\frac{\partial K_\theta}{\partial z}\right)}_{\mathscr{D}_{K_\theta}}$$

Constitutive relations for 1D

Stresses and fluxes are obtained:

$$\overline{uw} = f_1\left(\frac{\partial U}{\partial z}, \frac{\partial V}{\partial z}, \frac{\partial \Theta}{\partial z}, K, \varepsilon, K_\theta\right)$$

$$\overline{vw} = f_2\left(\frac{\partial U}{\partial z}, \frac{\partial V}{\partial z}, \frac{\partial \Theta}{\partial z}, K, \varepsilon, K_\theta\right)$$

$$\overline{w\theta} = f_3\left(\frac{\partial U}{\partial z}, \frac{\partial V}{\partial z}, \frac{\partial \Theta}{\partial z}, K, \varepsilon, K_\theta\right)$$

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[1]Želi, Velibor, Geert Brethouwer, Stefan Wallin, and Arne V. Johansson. "Consistent Boundary-Condition Treatment for Computation of the Atmospheric Boundary Layer Using the Explicit Algebraic Reynolds-Stress Model." *Boundary-Layer Meteorology* 171, no. 1 (April 15, 2019): 53–77. https://doi.org/10.1007/s10546-018-0415-x.

Constitutive relations

It's complicated[1]…

$$\beta_1 = -\frac{8}{15}\frac{N}{D_u}, \quad \beta_2 = 0, \quad \beta_3 = -\frac{8}{15}\frac{C_2 C_3 Q_6}{D_u Q_2}, \quad \beta_4 = -\frac{8}{15}\frac{C_2}{D_u},$$

$$\beta_{\theta 1} = \frac{2}{15}\frac{c_\Theta C_3 Q_3 Q_4}{D_u Q_1 Q_2}, \quad \beta_{\theta 2} = -\frac{16}{15}\frac{c_\Theta C_3}{D_u Q_1}\left(N_\theta \tilde{D}_u - c_\Theta C_3 N \gamma^{(\theta)}\right),$$

$$\beta_{\theta 3} = \frac{4}{15}\frac{c_\Theta C_2 C_3 N_\theta Q_4}{D_u Q_1 Q_2}, \quad \beta_{\theta 4} = -\frac{32}{15}\frac{c_\Theta C_2 C_3}{D_u Q_1}\left(N N_\theta + Q_3\right),$$

$$\beta_{g1} = \frac{8}{15}\frac{C_3 Q_3 Q_5}{D_u Q_1 Q_2}, \quad \beta_{g2} = \frac{8}{15}\frac{C_2 C_3 N_\theta Q_5}{D_u Q_1 Q_2},$$

but there is physics behind the madness.

$$\lambda_{\theta 1} = -\frac{2}{15}\frac{c_\Theta Q_4}{D_u Q_1 Q_2}\left(N_\theta D_u - c_\Theta C_3 N \gamma^{(\theta)}\right), \quad \lambda_{\theta 2} = \frac{16}{15}\frac{c_\Theta Q_3}{Q_1},$$

$$\lambda_{\theta 3} = \frac{4}{15}\frac{c_\Theta^2 C_2 C_3 Q_4 \gamma^{(\theta)}}{D_u Q_1 Q_2}, \quad \lambda_{\theta 4} = \frac{32}{15}\frac{c_\Theta C_2 N_\theta}{Q_1},$$

$$\lambda_{g1} = -\frac{4}{15}\frac{Q_5}{D_u Q_1 Q_2}\left(N_\theta D_u - c_\Theta C_3 N \gamma^{(\theta)}\right), \quad \lambda_{g2} = 0,$$

$$\lambda_{g3} = \frac{8}{15}\frac{c_\Theta C_2 C_3 Q_5 \gamma^{(\theta)}}{D_u Q_1 Q_2}, \quad \lambda_{g4} = 0, \qquad (36)$$

where

$$D_u = N^2 - 2C_2^2 II_\Omega \geq 0, \quad \tilde{D}_u = N^2 + 2C_2^2 II_\Omega,$$

$$Q_1 = 2Q_3^2 - 4C_2^2 II_\Omega N_\theta^2 \geq 0,$$

$$Q_2 = 6N D_u N_\theta^2 - 2c_\Theta C_3 \gamma^{(\theta)} N_\theta (6N^2 + D_u) + 8c_\Theta^2 C_3^2 N \gamma^{(\theta)2},$$

$$Q_3 = N N_\theta - c_\Theta C_3 \gamma^{(\theta)},$$

$$Q_4 = Q_2(15D_u + 8C_2 II_\Omega) + 2c_\Theta C_3 \gamma^{(\theta)} N_\theta D_u (15D_u + 16C_2 II_\Omega) - 6D_u N N_\theta^2 (5D_u + 8C_2 II_\Omega) - 60c_\Gamma C_3 D_u \Gamma^2 (N Q_3 + C_2^2 II_\Omega N_\theta),$$

$$Q_5 = 15c_\Gamma c_\Theta^2 C_3^2 N D_u \gamma^{(\theta)2} - 30c_\Gamma c_\Theta C_3 \gamma^{(\theta)} N_\theta D_u (2N^2 - C_2^2 II_\Omega) + 45c_\Gamma N N_\theta^2 D_u^2 + 2c_\Theta^2 C_3 N \Theta^2 \left(3N N_\theta (5D_u + 8C_2 II_\Omega) - c_\Theta C_3 \gamma^{(\theta)} (15D_u + 16C_2 II_\Omega)\right),$$

$$Q_6 = -16C_3 N c_\Theta^2 \gamma^{(\theta)2} + 6c_\Theta N_\theta \gamma^{(\theta)}\left(5C_2 D_u + 2\tilde{D}_u\right) + 45c_\Gamma C_2 N_\theta \Gamma^2 D_u, \qquad (37)$$

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EARSM in practice

Complicated to derive, but easy to use.

$$S_{ij} = \frac{\tau}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$$

$$\Omega_{ij} = \frac{\tau}{2}\left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}\right)$$

$$\Theta_i = \tau\sqrt{\frac{K}{K_\theta}}\frac{\partial \Theta}{\partial x_i}$$

$$\Gamma_i = \tau\sqrt{\frac{K_\theta}{K}}\frac{g_i}{T_0}$$

→

Invariants

$$I\!I_S, I\!I_\Omega, \Theta^2, \Gamma^2, \gamma^{(\theta)}, \gamma_S^{(\theta)}, \gamma_\Omega^{(\theta)}, \gamma_{S\Omega}^{(\theta)}$$

E.g. $I\!I_S \equiv S_{ij}S_{ji}$

→

Non-linearities

Solve cubic equations for N and N_θ

→

Coefficients

$$\beta_1, \ldots, \beta_{10}$$
$$\lambda_1, \ldots, \lambda_8$$

→

Expansion

$$a_{ij} = \sum_{k=1}^{10} \beta_k T_{ij}^{(k)}$$

$$\xi_i = \sum_{k=1}^{8} \lambda_k V_i^{(k)}$$

E.g. $T_{ij}^{(1)} \equiv S_{ij}$

$V_i^{(1)} \equiv \Theta_i$

→

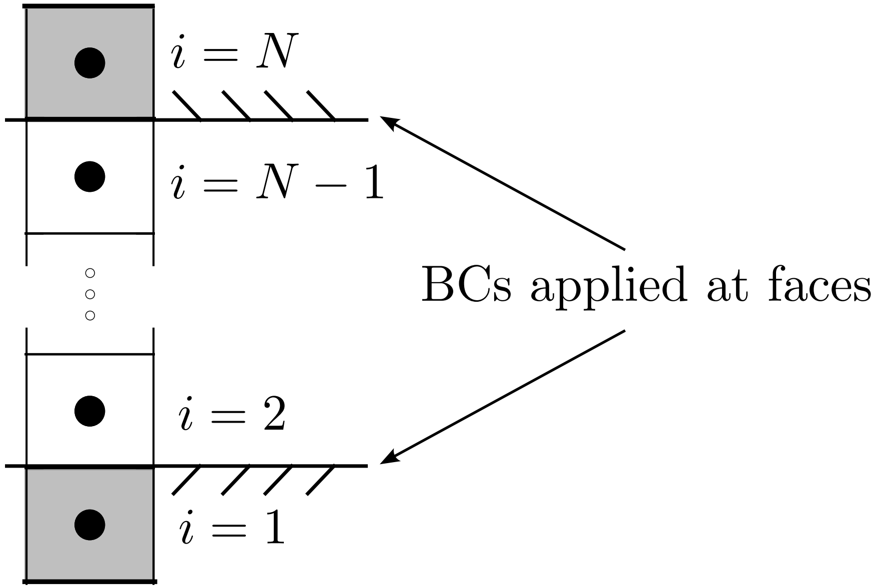
Result

$$\overline{u_i u_j} = a_{ij}K + \frac{2}{3}K\delta_{ij}$$

$$\overline{u_i\theta} = \xi_i\sqrt{KK_\theta}$$

Numerical solver

- Python 1d-solver
- General 1D transient PDE solver
- Implicit Euler time-stepping
- Central difference scheme
- Grid and time stretching possible
- Equations and BCs defined symbolically




```
eqd = {'U':   'Diff(U,t) - Diff(nut*Diff(U,y),y) = Px',
       'K':   'Diff(K,t) - Diff(nut/sigk*Diff(K,y),y) + fe*K = Pk',
       'eps': 'Diff(eps,t) - Diff(nut/sige*Diff(eps,y),y) + Ce2*fe*eps = Ce1*Pk*fe',
       }
BCd = {'U':   ['U = 0.0', 'Diff(U,y) = 0'],
       'K':   ['K = utau**2/sqrt(Cmu)', 'Diff(K,y) = 0'],
       'eps': ['eps = utau**3/(kappa*y0)', 'Diff(eps,y) = 0']
       }
eqt = {'utau':   'utau = kappa*sqrt(U_[1]**2.0)/(ln(yc_[1]/y0+1.0))'
       }
fyd = {'sqrt': sqrt,
       'ln': ln,
```

Case description

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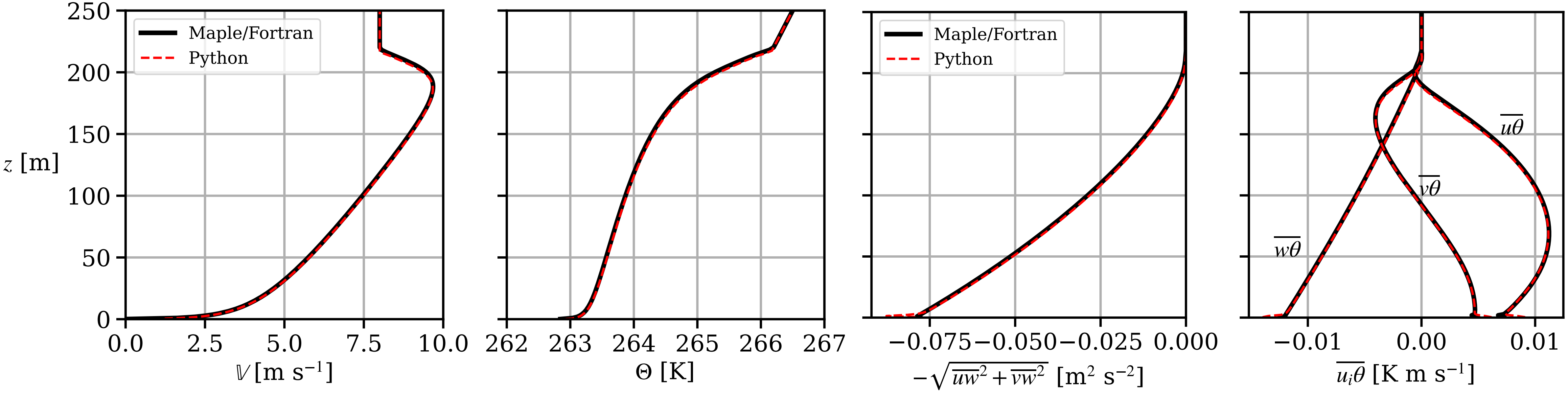
[1]Lazeroms, W. M.J., G. Svensson, E. Bazile, G. Brethouwer, S. Wallin, and A. V. Johansson. "Study of Transitions in the Atmospheric Boundary Layer Using Explicit Algebraic Turbulence Models." *Boundary-Layer Meteorology* 161, no. 1 (2016): 19–47. https://doi.org/10.1007/s10546-016-0194-1.

[2]Želi, Velibor, Geert Brethouwer, Stefan Wallin, and Arne V. Johansson. "Consistent Boundary-Condition Treatment for Computation of the Atmospheric Boundary Layer Using the Explicit Algebraic Reynolds-Stress Model." *Boundary-Layer Meteorology* 171, no. 1 (April 15, 2019): 53–77. https://doi.org/10.1007/s10546-018-0415-x.

[3]Želi, Velibor, Geert Brethouwer, Stefan Wallin, and Arne V. Johansson. "Modelling of Stably Stratified Atmospheric Boundary Layers with Varying Stratifications." *Boundary-Layer Meteorology* 176, no. 2 (August 1, 2020): 229–49. https://doi.org/10.1007/s10546-020-00527-8.

Verification

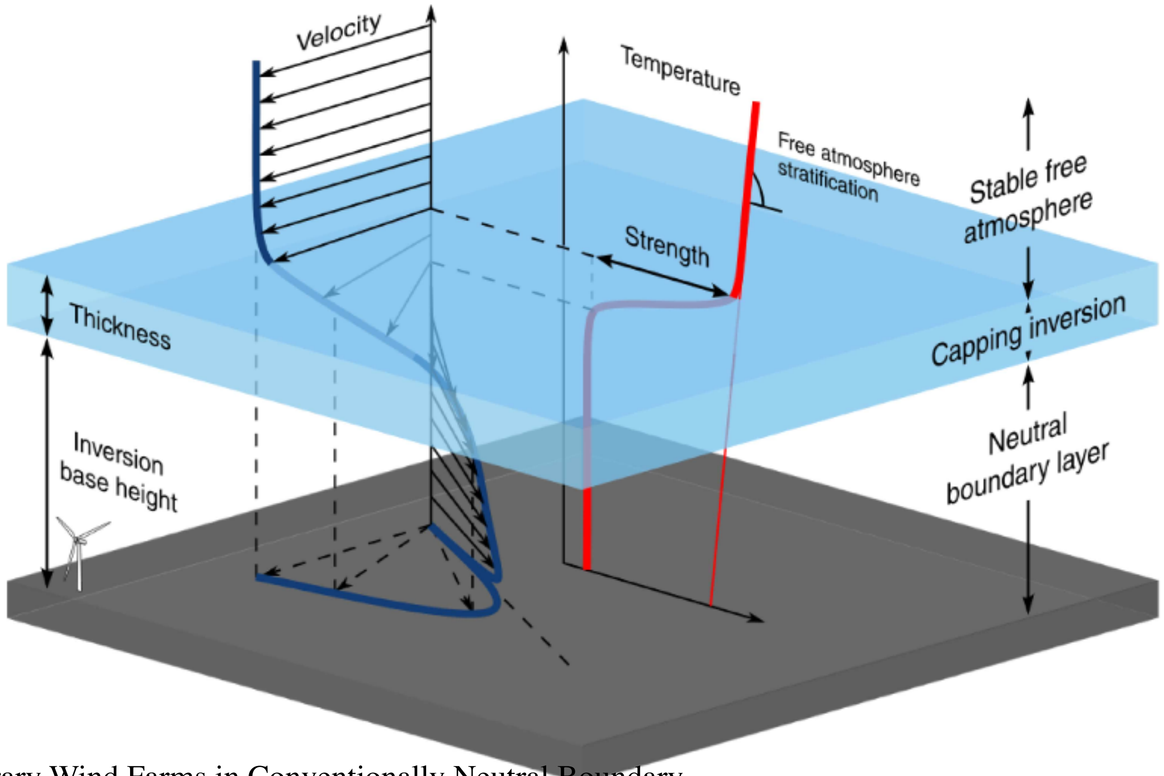
- Previous studies[1,2,3] used a Maple-Fortran code
- Verify correctness of Python 1d-solver with GABLS1 case[3]



[1]Allaerts, Dries, and Johan Meyers. “Large Eddy Simulation of a Large Wind-Turbine Array in a Conventionally Neutral Atmospheric Boundary Layer.” *Physics of Fluids* 27, no. 6 (2015): 065108. https://doi.org/10.1063/1.4922339.
[2]Lanzilao, L., and J. Meyers. “A Parametric Large-Eddy Simulation Study of Wind-Farm Blockage and Gravity Waves in Conventionally Neutral Boundary Layers.” *Journal of Fluid Mechanics* 979 (2024): A54. https://doi.org/10.1017/jfm.2023.1088.

Case study: conventionally neutral boundary layer (CNBL)

- Popular ABL type for wind farm simulations[1,2,3,4].
- The buoyant EARSM has not been tested for CNBLs yet.

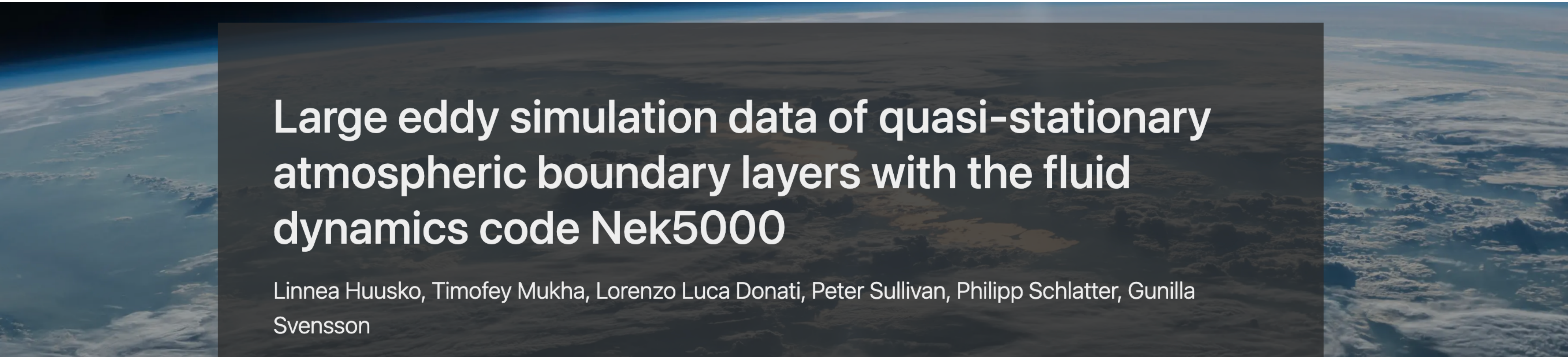


[3]Stipa, Sebastiano, D. Allaerts, and J. Brinkerhoff. “A Shear Stress Parametrization for Arbitrary Wind Farms in Conventionally Neutral Boundary Layers.” *Journal of Fluid Mechanics* 981 (2024): 1–15. https://doi.org/10.1017/jfm.2024.22.
[4]Ghobrial, Mina, Tim Stallard, David M. Schultz, and Pablo Ouro. “Evaluation of Six Subgrid-Scale Models for LES of Wind Farms in Stable and Conventionally-Neutral Atmospheric Stratification.” *Boundary-Layer Meteorology* 191, no. 4 (2025): 19. https://doi.org/10.1007/s10546-025-00907-y.

Huusko, L, T Mukha, L L Donati, P P Sullivan, and P Schlatter. “Large Eddy Simulation of Canonical Atmospheric Boundary Layer Flows with the Spectral Element Method in Nek5000.” *Journal of Advances in Modeling Earth Systems*, ahead of print, 2025. https://doi.org/10.22541/essoar.174793542.24700933/v1.

CNBL reference data

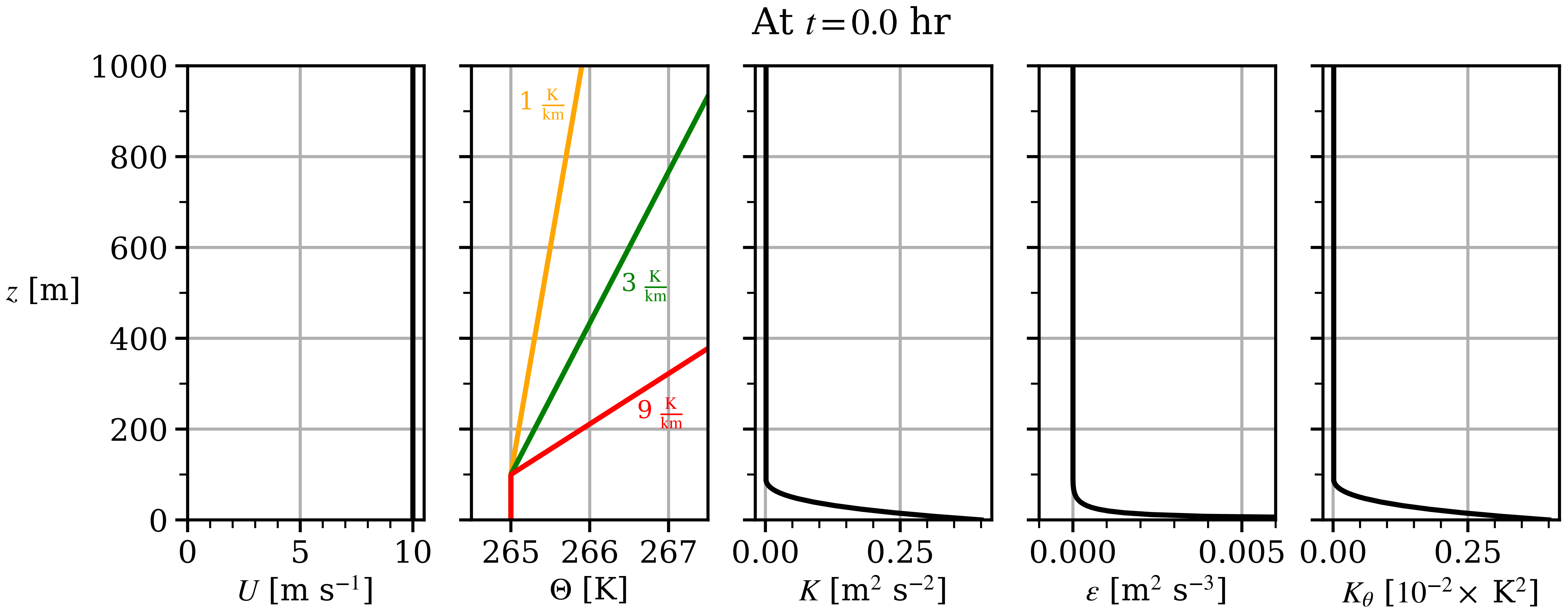
- Reference LES data from Stockholm University and NCAR.
- Two codes (Nek5000 and NCAR spectral)



https://bolin.su.se/data/huusko-2025-les-nek5000-1

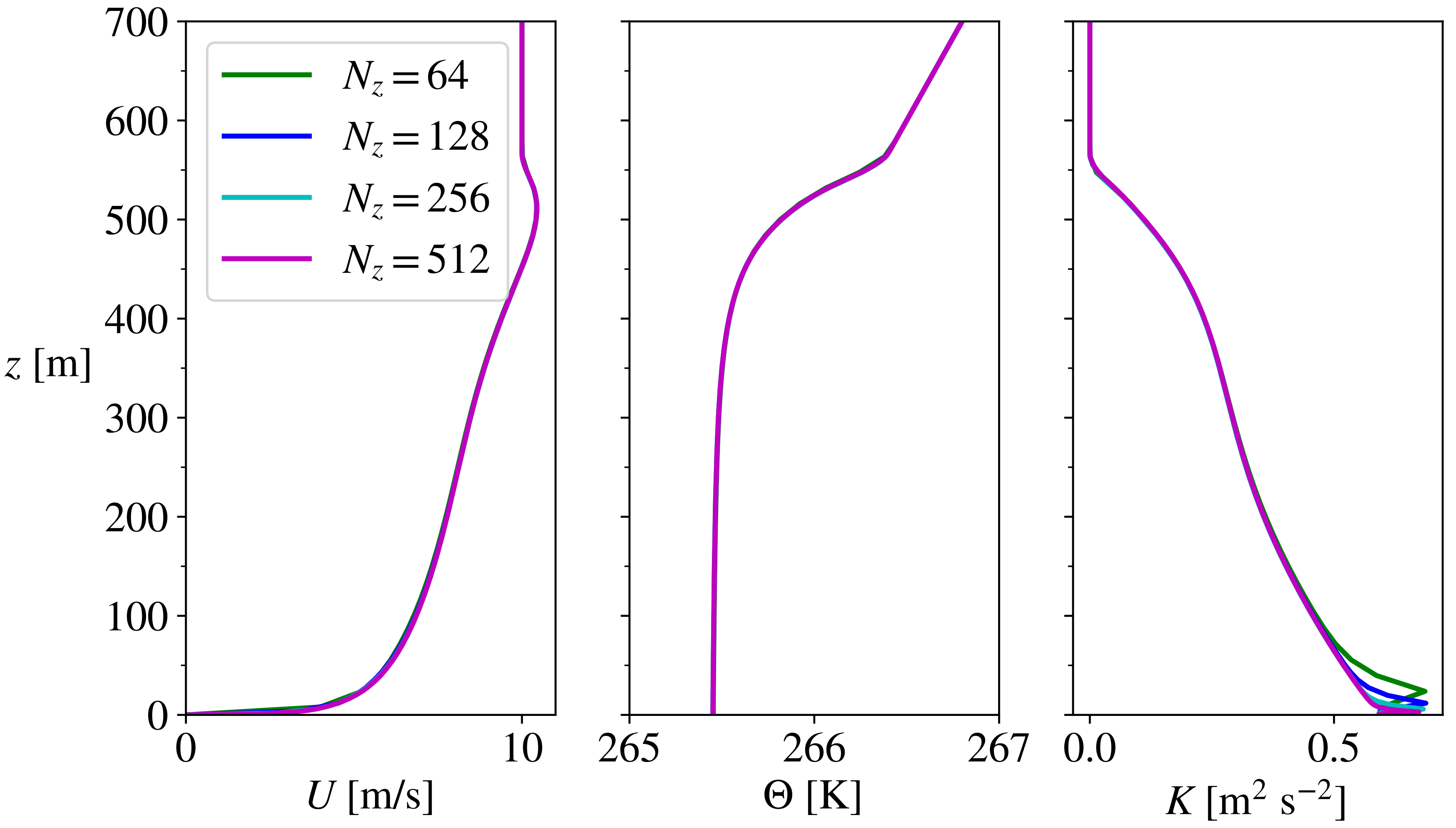
CNBL setup

- $G = 10$ m/s
- $z_0 = 0.1$ m
- $f_c = 10^{-4}$ s^{-1}
- $\Gamma = \{1,3,9\}$ K/km
- $N_z = 256$
- $\Delta z \approx 3.9$ m
- $\Delta t = 60$ s
- $t_{tot} \approx 10$ hr
- $t_{stat} \approx [8.3, 10]$ hr



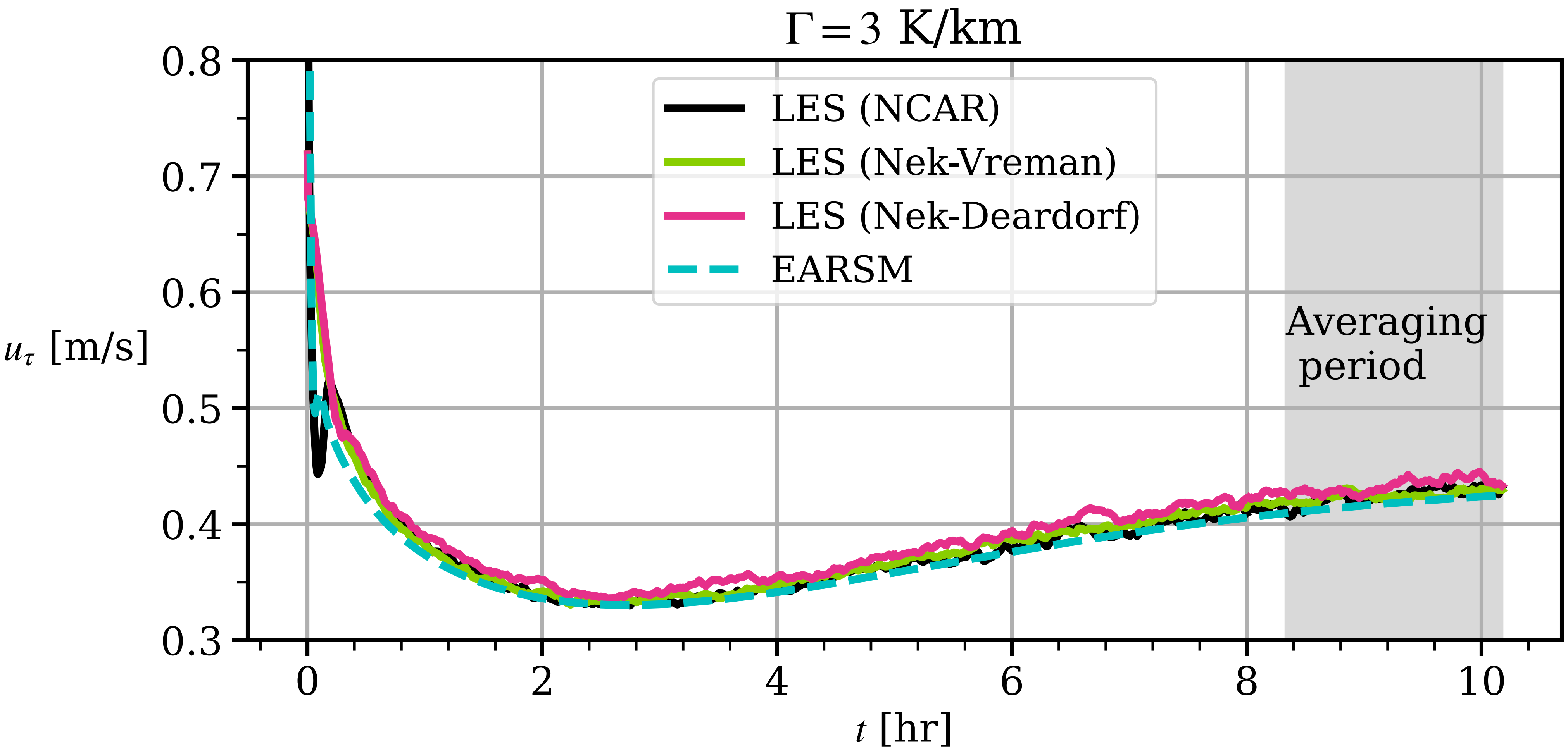
Grid convergence

$\Gamma = 3$ K/km case

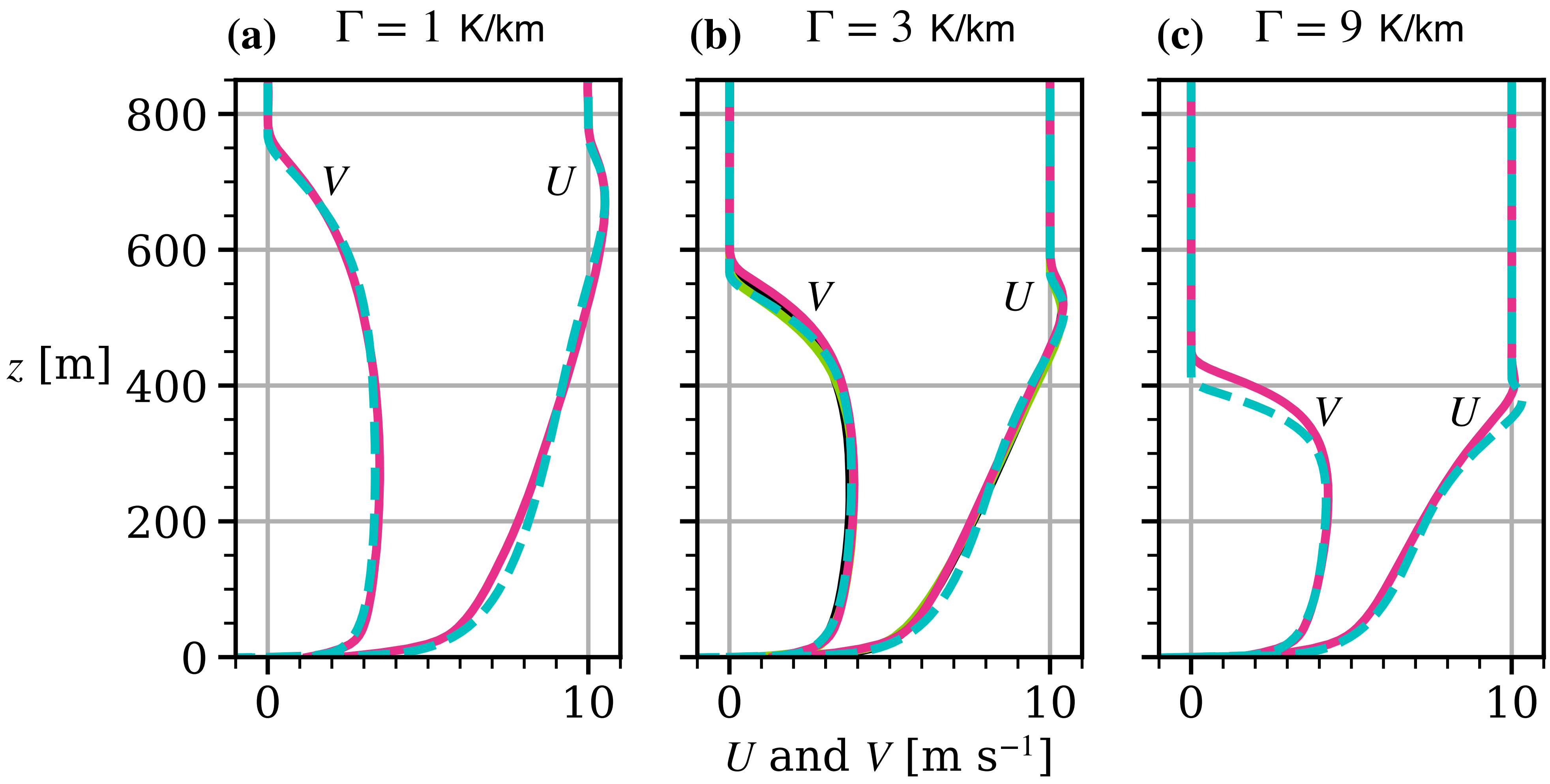


Results (velocity)

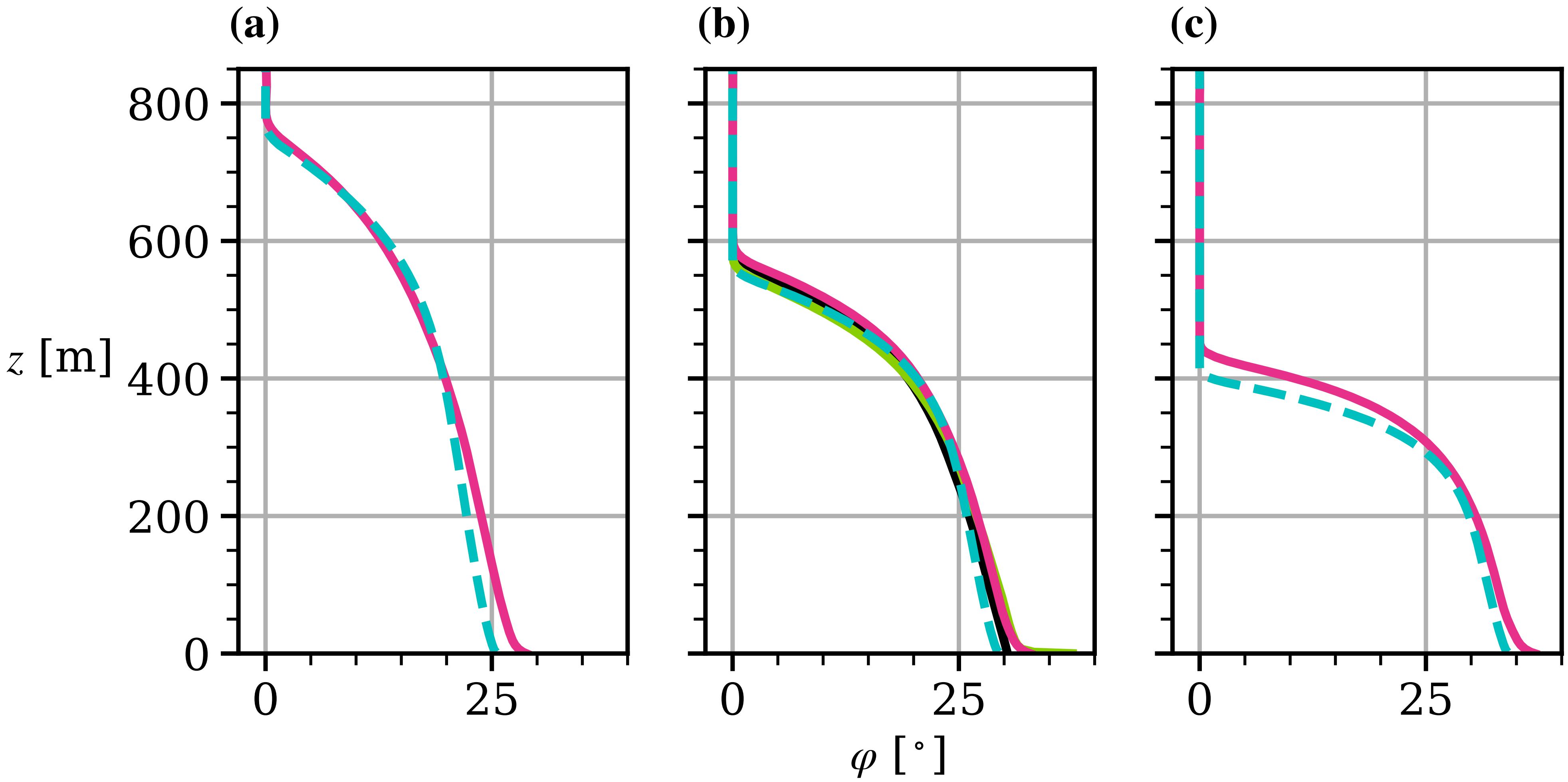
Time evolution of friction velocity



Velocity components



Wind direction



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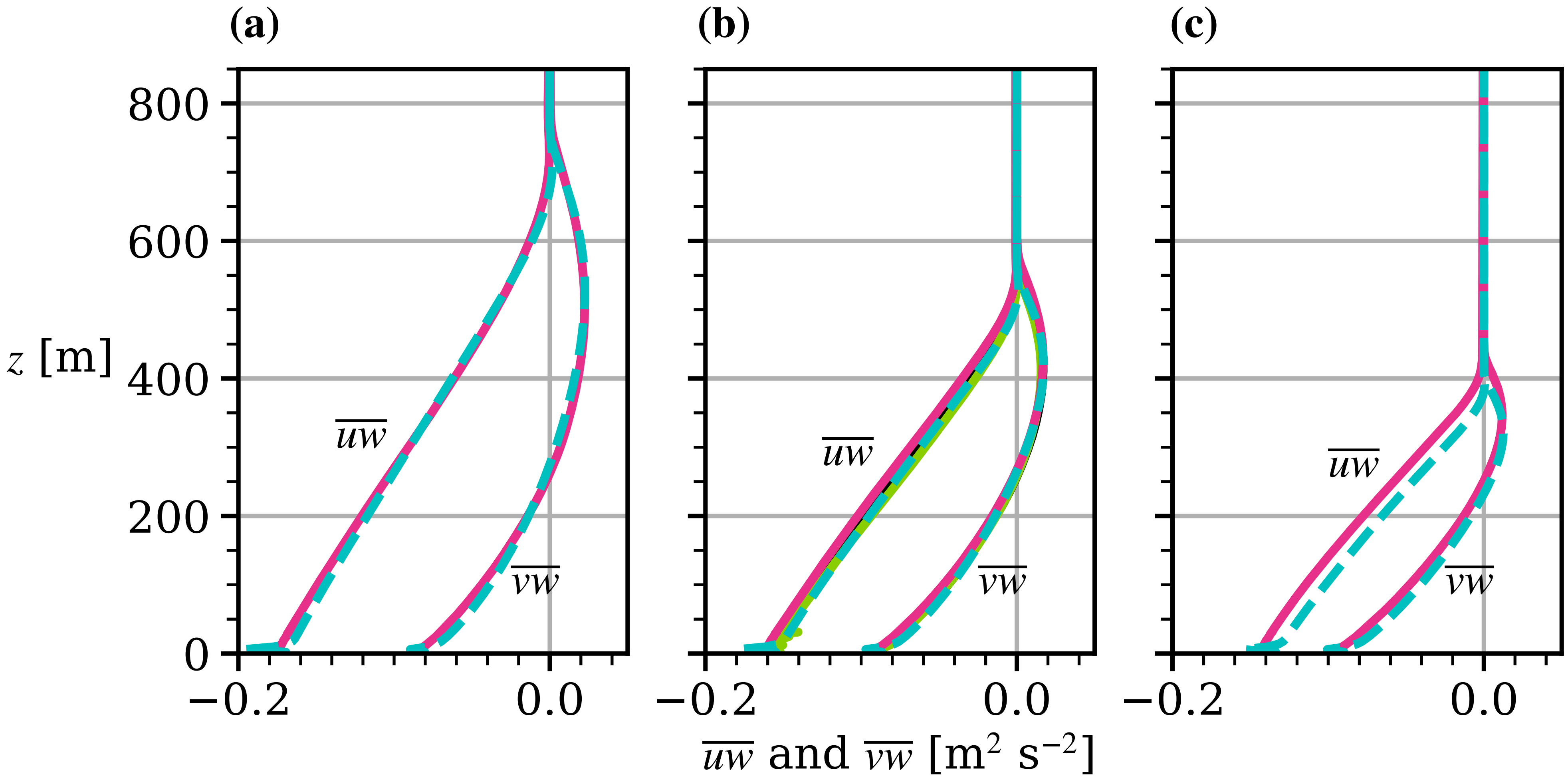
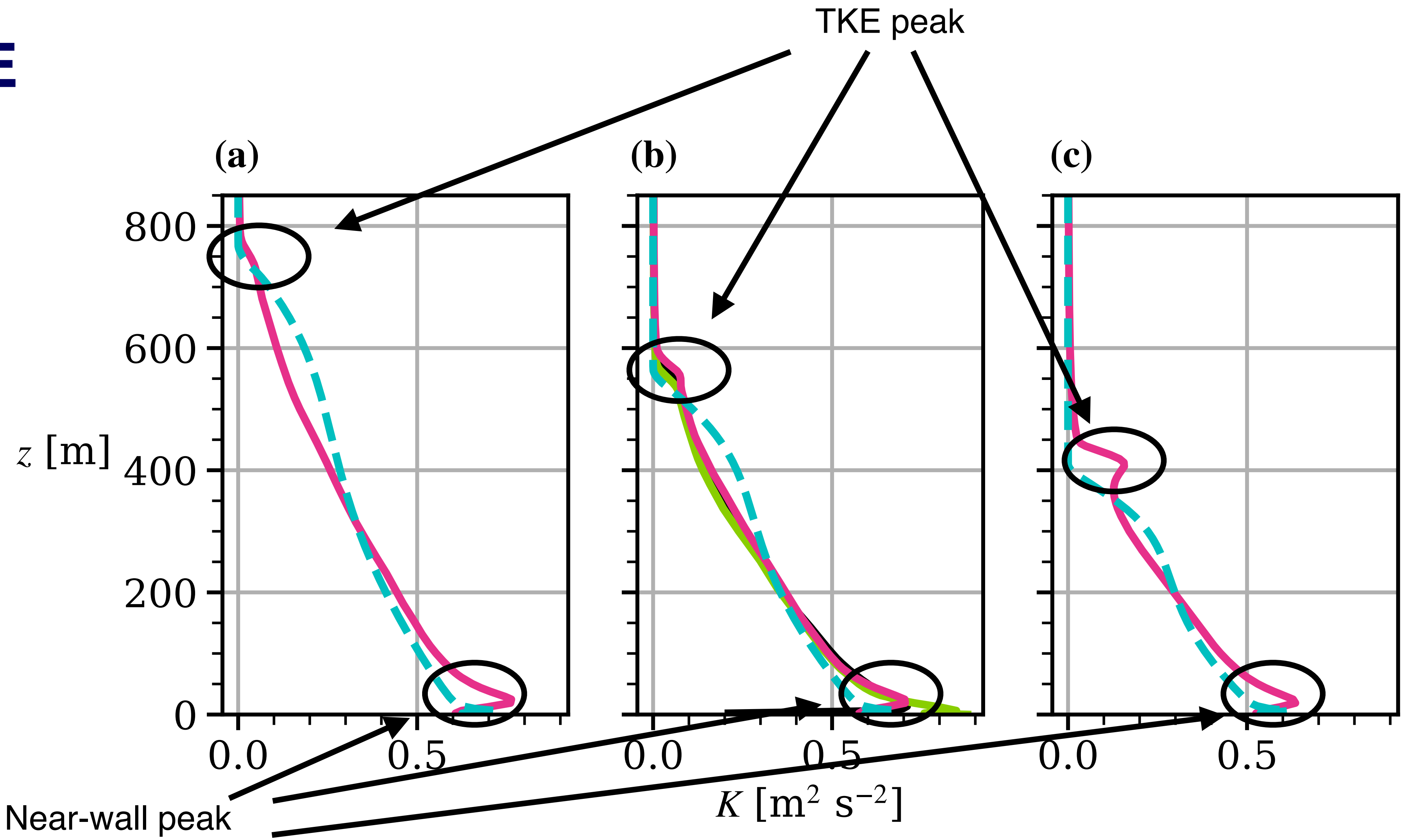
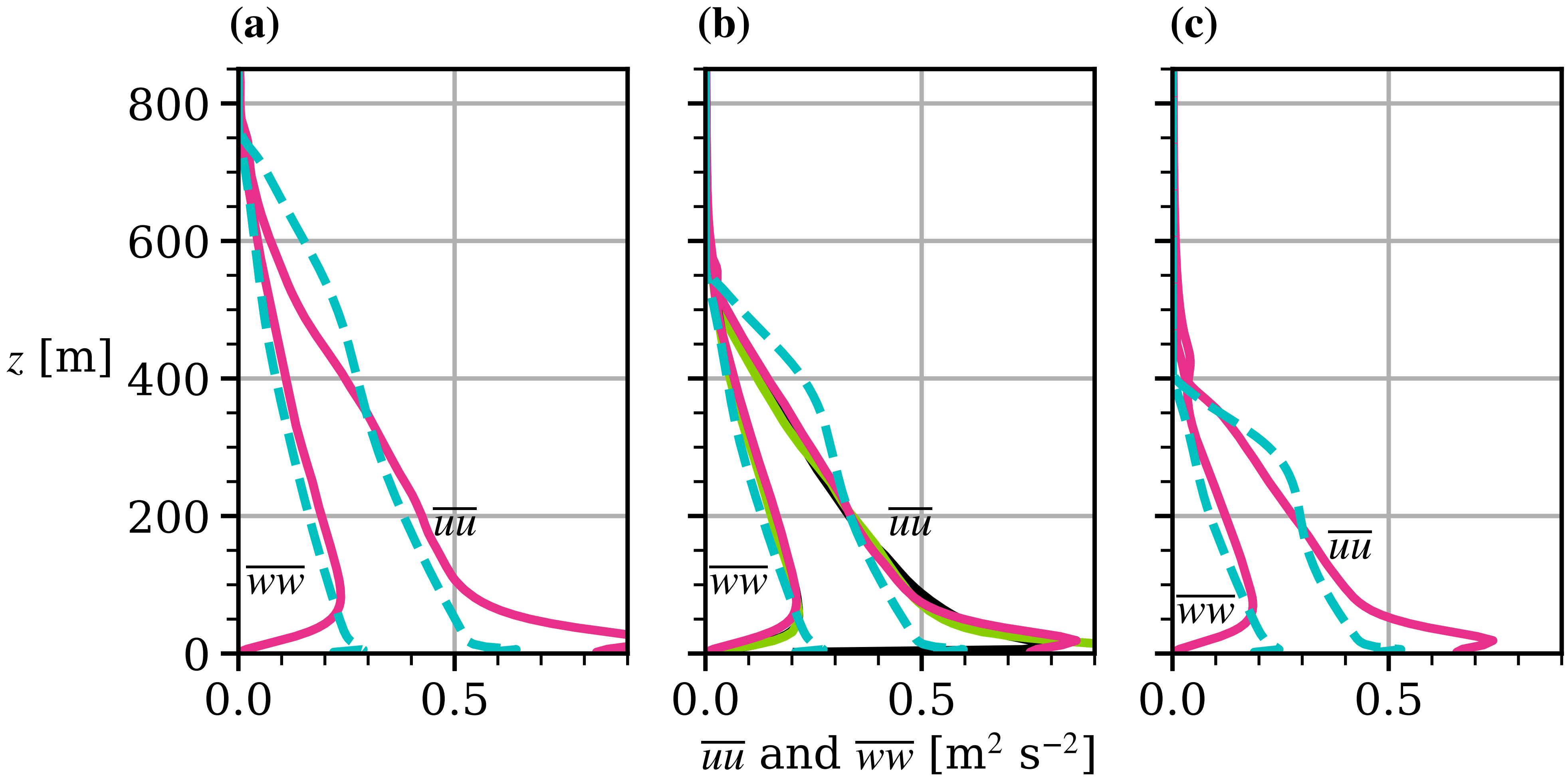
Shear stresses



TKE



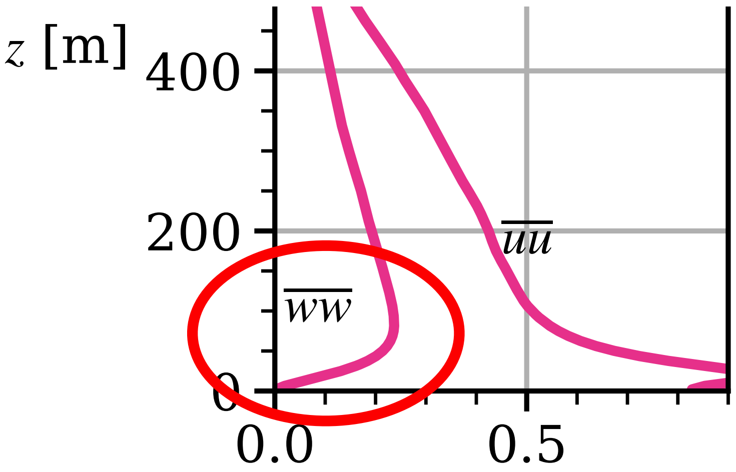
Streamwise and vertical velocity variances



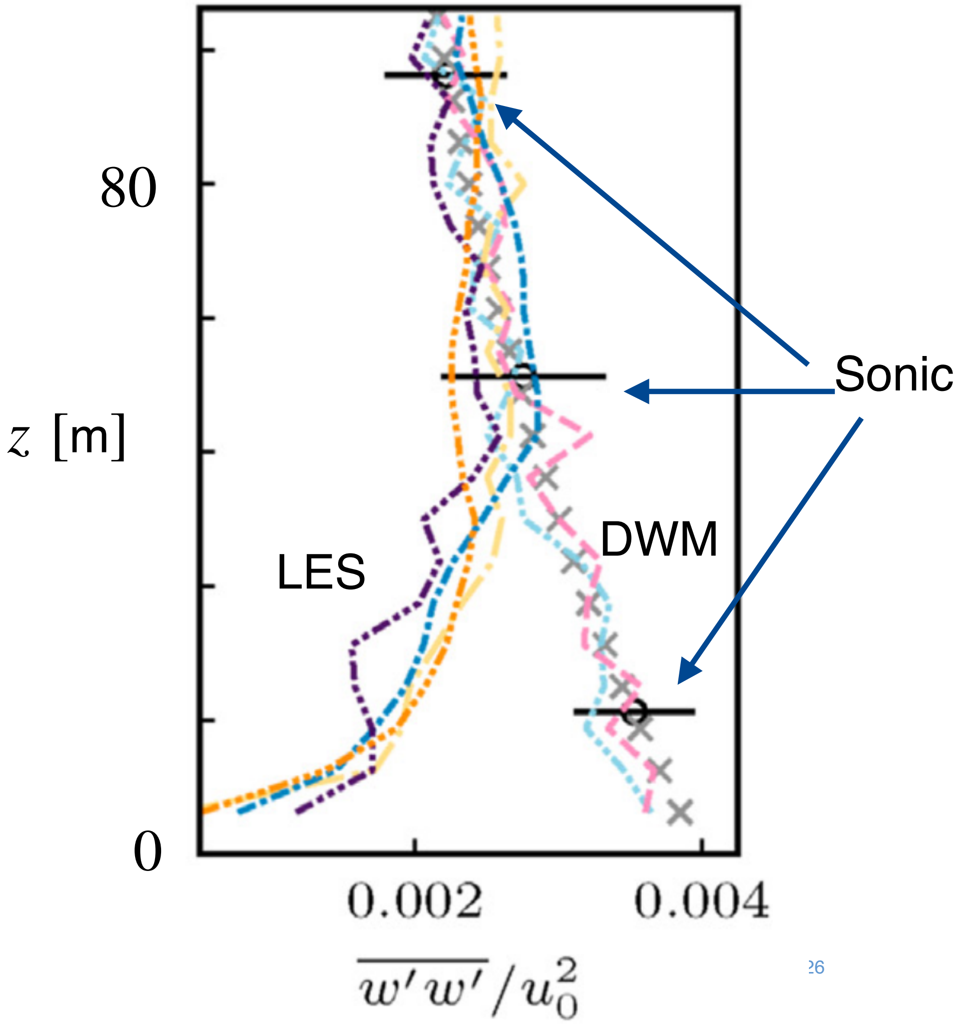
[1]Asmuth, Henrik, Gonzalo P. Navarro Diaz, Helge Aagaard Madsen, et al. "Wind Turbine Response in Waked Inflow: A Modelling Benchmark against Full-Scale Measurements." *Renewable Energy* 191 (May 2022): 868–87. https://doi.org/10.1016/j.renene.2022.04.047.
[2]Wyngaard, John C. *Turbulence in the Atmosphere*. In *Turbulence in the Atmosphere*. Cambridge University Press, 2010. https://doi.org/10.1017/CBO9780511840524.

Experimental data: Vertical velocity variance

- Vertical velocity variance, $\overline{ww}$, should *not* go to zero in the surface layer.
- Classic surface layer theory[2]: $\left.\frac{\overline{ww}}{u_*^2}\right|_{z/L=0} \approx 1.2$

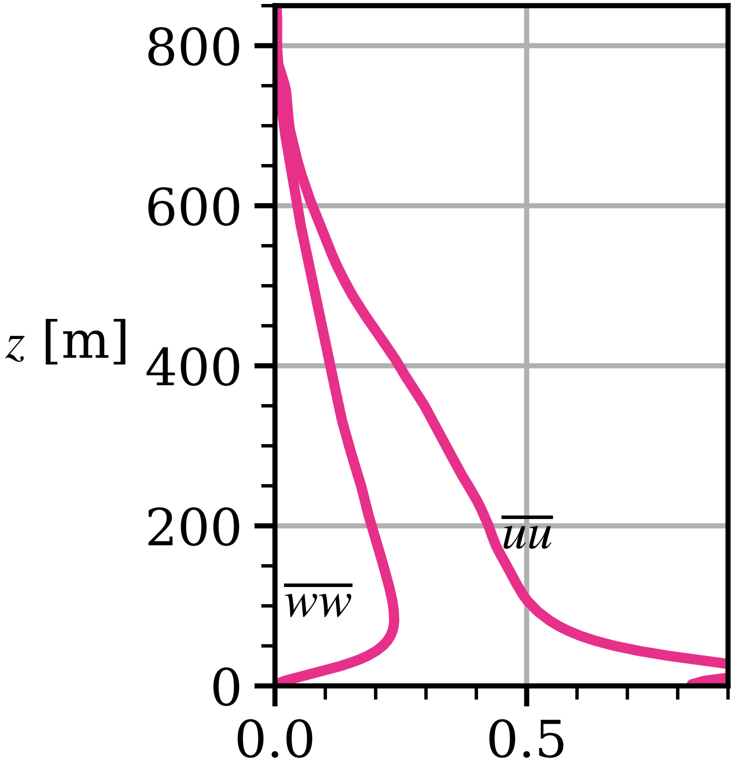


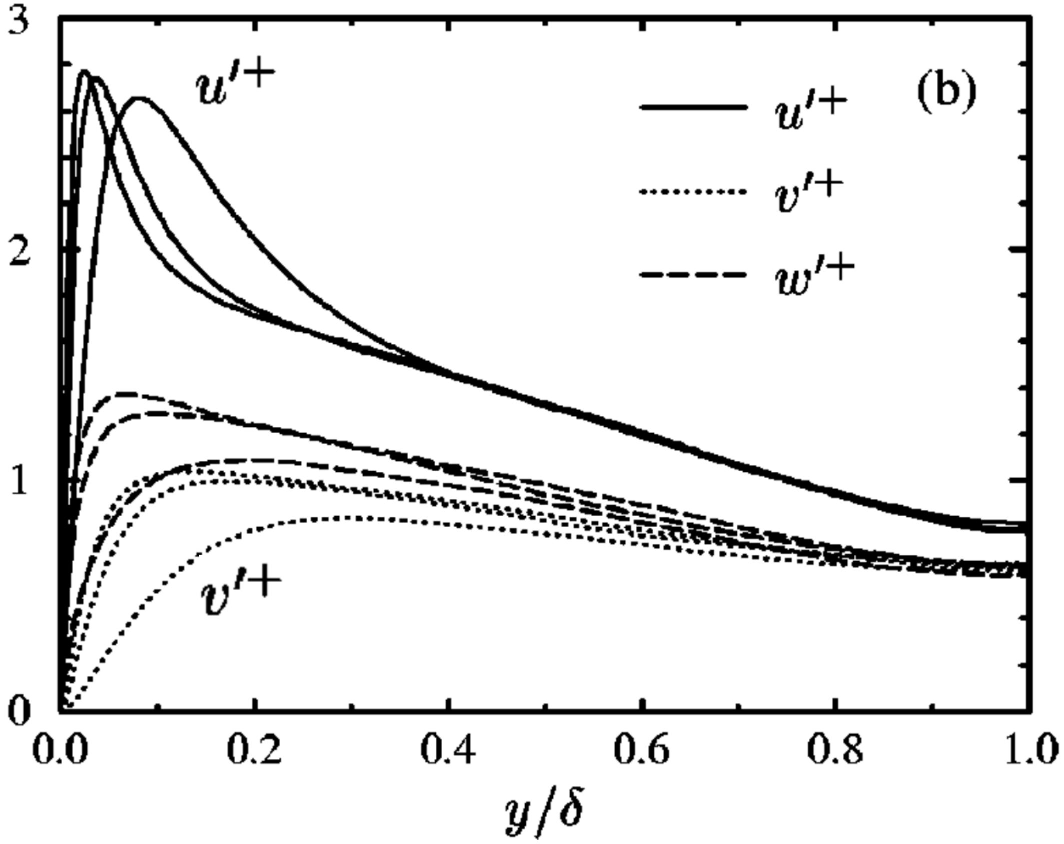
Data from Asmuth et al. (2022)[1]



Analogy with low-Re DNS

Atmospheric LES (very high-Re) behaves as a low-Re DNS[1]!





[1]Moser, Robert D., John Kim, and Nagi N. Mansour. "Direct Numerical Simulation of Turbulent Channel Flow up to Reτ=590." *Physics of Fluids* 11, no. 4 (1999): 943–45. https://doi.org/10.1063/1.869966.

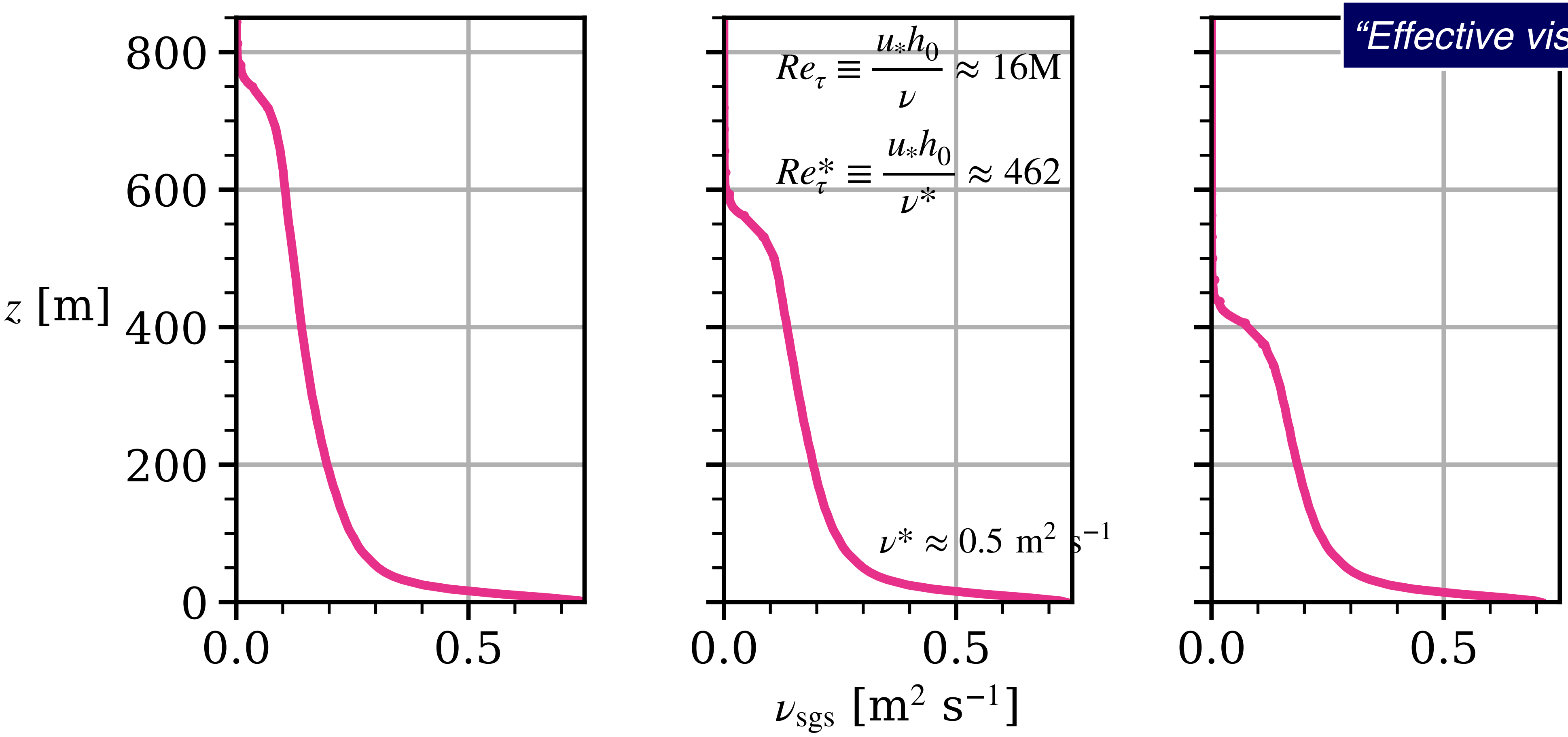
$\nu \approx 1.5 \cdot 10^{-5}\ \mathrm{m^2\ s^{-1}}$
$u_* \approx 0.42\ \mathrm{m\ s^{-1}}$

SGS viscsosity

"LES is DNS of a LES fluid"

$$\frac{D\tilde{u}_i}{Dt} = -\frac{1}{\rho}\frac{\tilde{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left((\nu + \nu_{sgs})\frac{\partial \tilde{u}_i}{\partial x_j}\right) + \tilde{f}_i$$

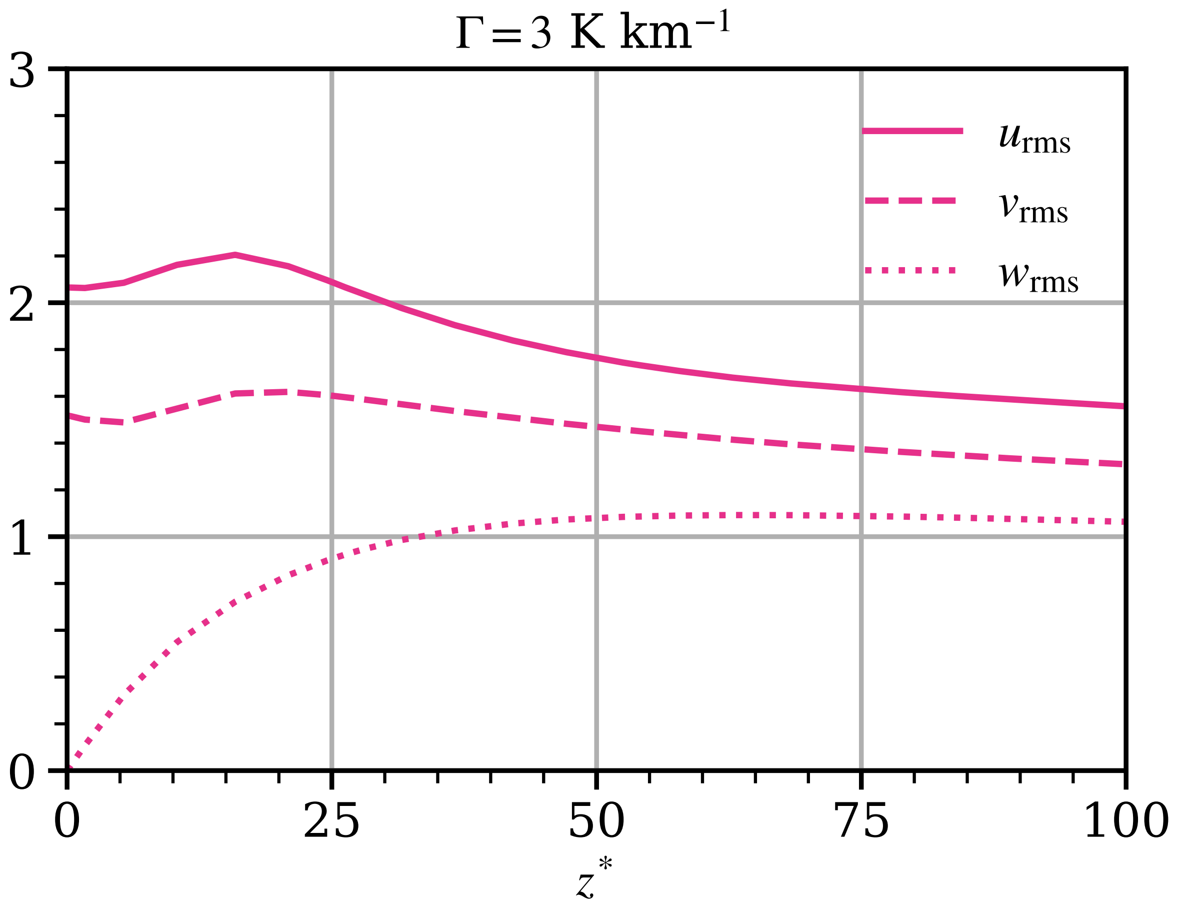
"Effective viscosity", ν^*

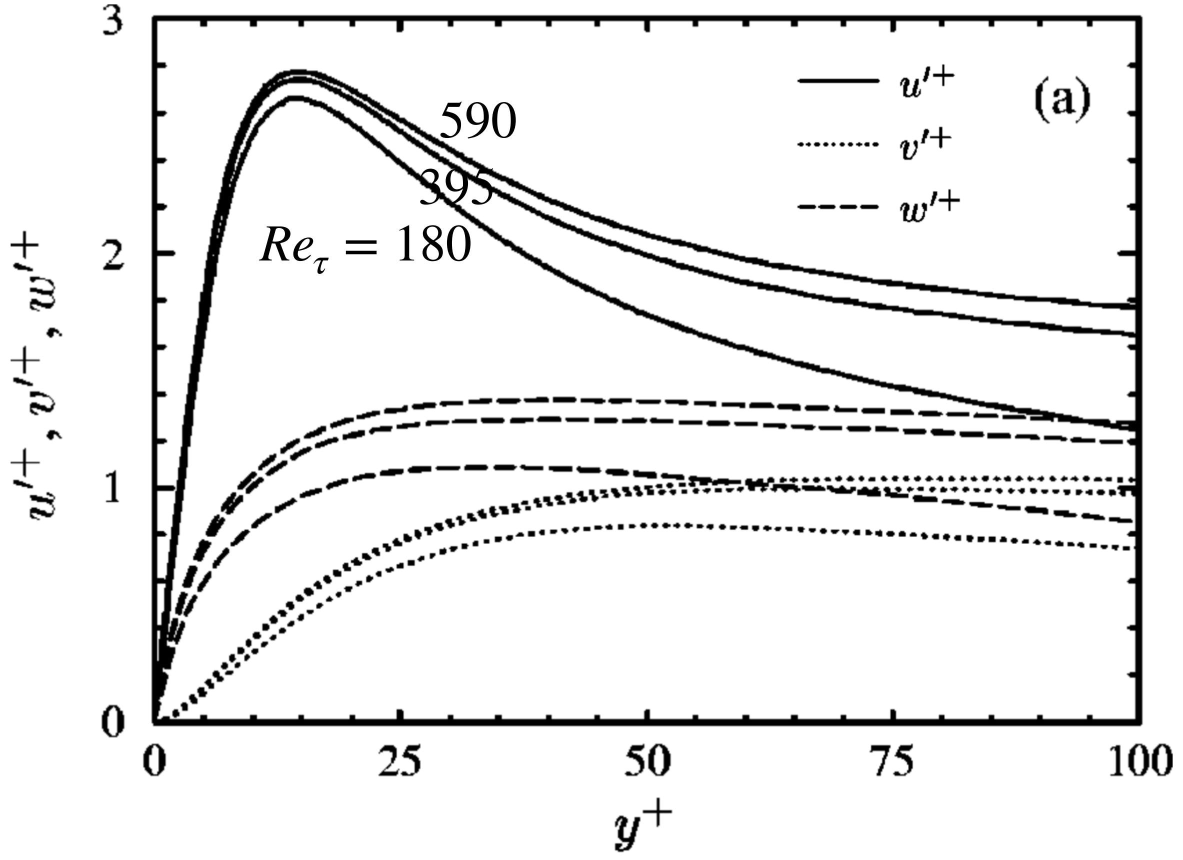


[1]Moser, Robert D., John Kim, and Nagi N. Mansour. "Direct Numerical Simulation of Turbulent Channel Flow up to Reτ=590." *Physics of Fluids* 11, no. 4 (1999): 943–45. https://doi.org/10.1063/1.869966.

$$\nu^* \approx 0.5 \text{ m}^2 \text{ s}^{-1} \quad u_* \approx 0.42 \text{ m s}^{-1} \quad \Rightarrow \quad \delta_v^* \equiv \frac{\nu^*}{u_*} \approx 1.2 \text{ m}$$

Viscous coordinate, $z^* \equiv z/\delta_v^*$

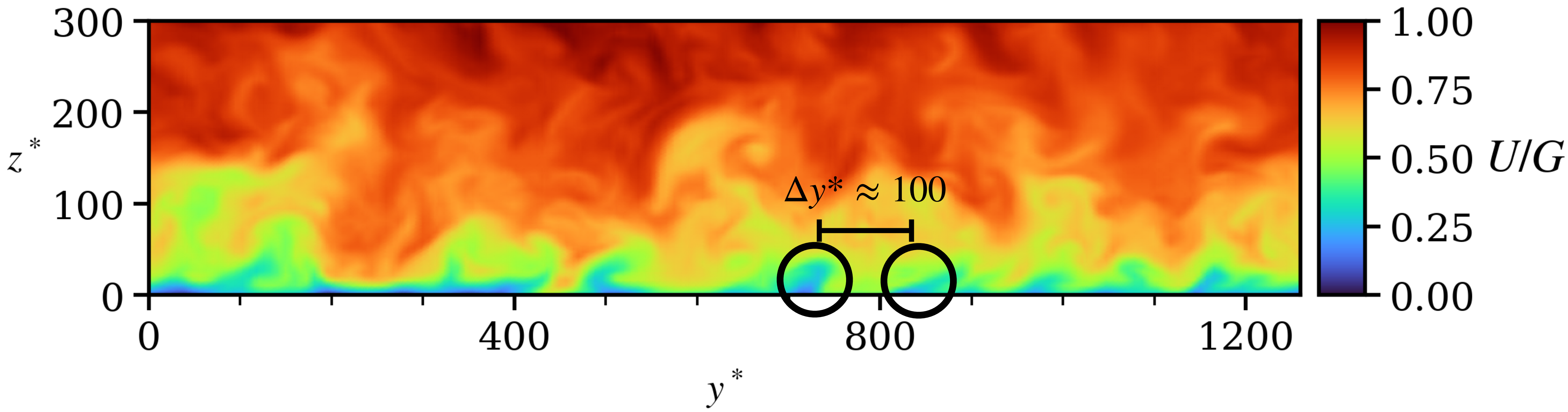




Robinson, S K. "Coherent Motions in the Turbulent Boundary Layer." *Annual Review of Fluid Mechanics* 23, no. 1 (1991): 601–39. https://doi.org/10.1146/annurev.fl.23.010191.003125.

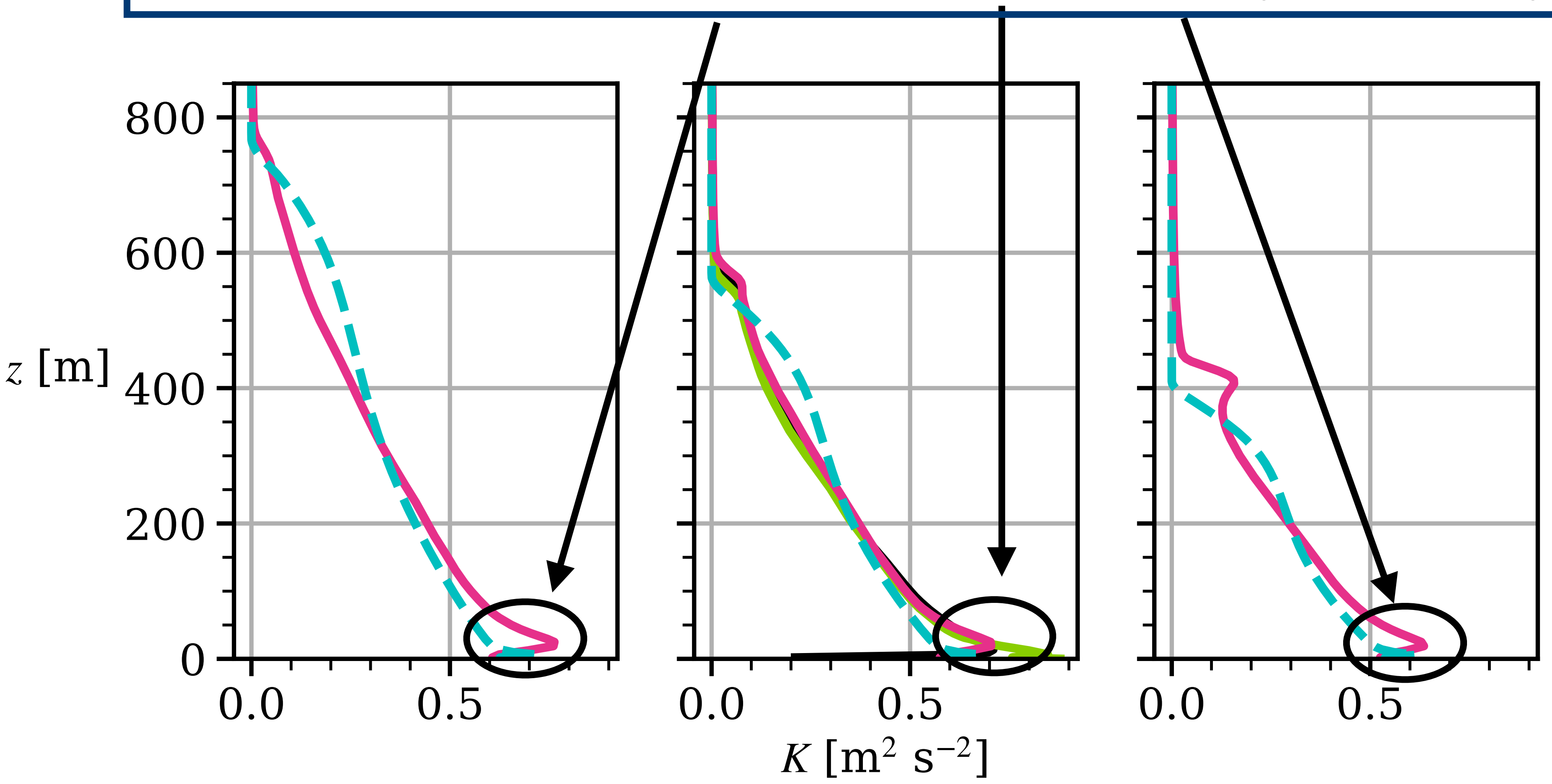
Another evidence: Streaks in the near-wall layer

- Elongated structures of high- and low-speed streamwise velocity (Robinson 1991) exist in the buffer layer of low-Re flows.
- Universal mean spanwise spacing of approximately 100 viscous lengths (Robinson 1991).



Near-wall TKE peak conclusion

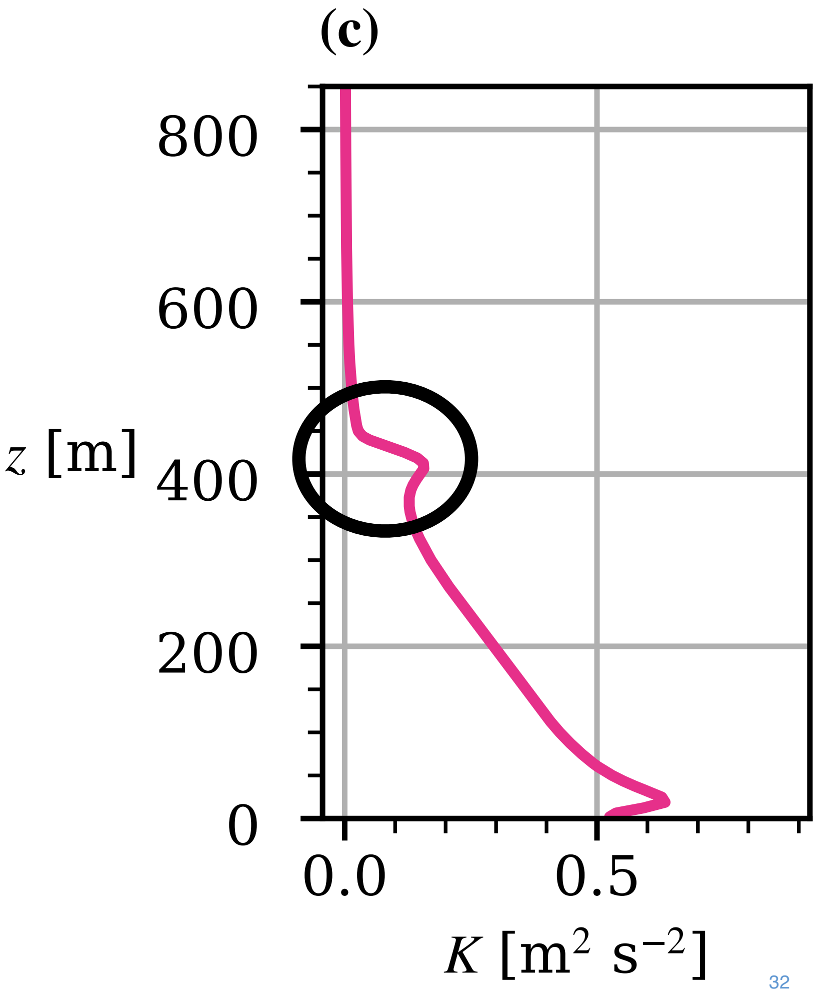
Near-wall TKE peak in LES comes from artificial buffer layer → Not physical!



TKE peak in inversion layer

Why is there a peak in the LES data?

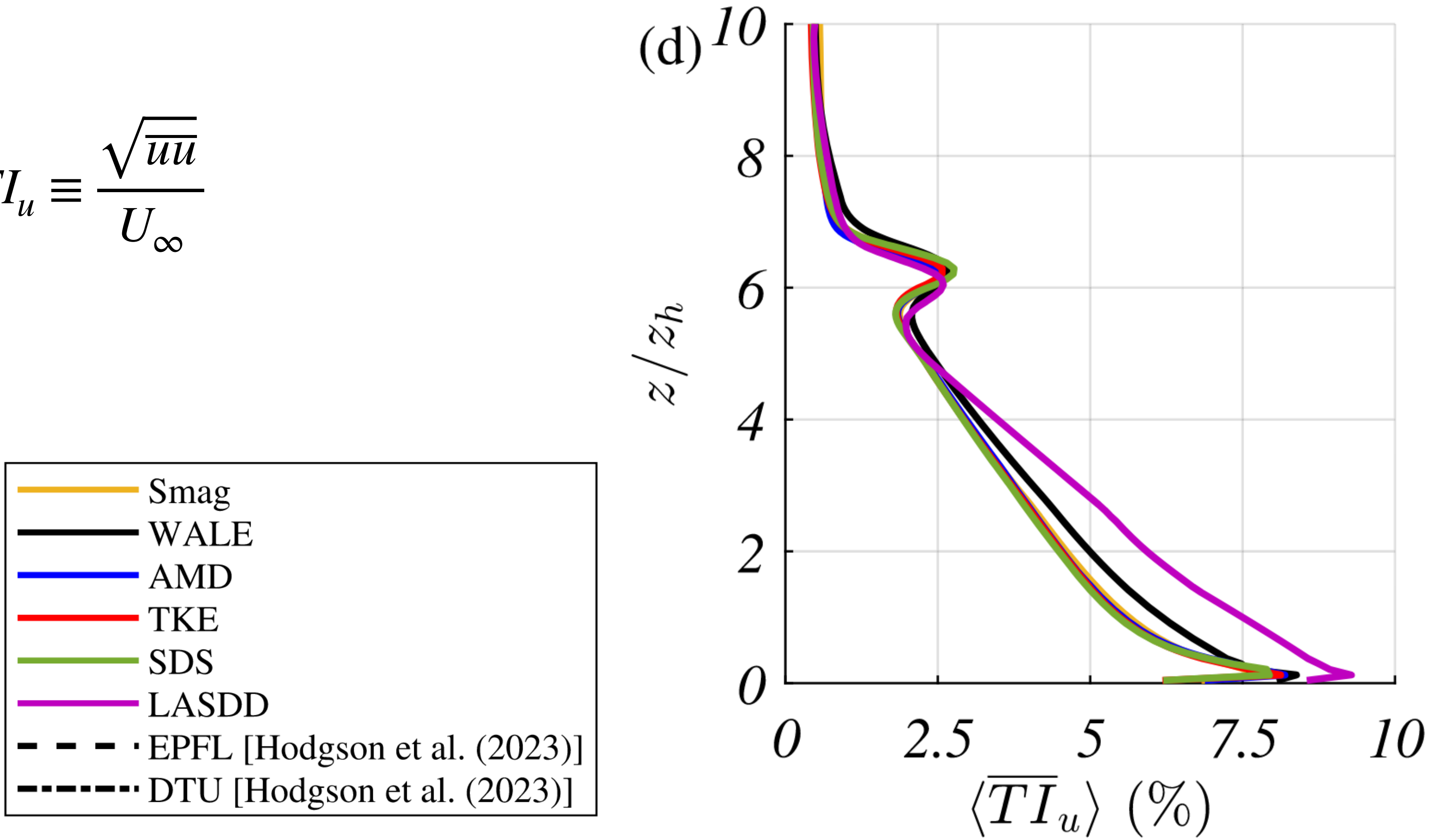
- Code?
- SGS modelling?
- Insufficient resolution?
- Slow non-turbulent oscillation?
- …



[1]Ghobrial, Mina, Tim Stallard, David M. Schultz, and Pablo Ouro. "Evaluation of Six Subgrid-Scale Models for LES of Wind Farms in Stable and Conventionally-Neutral Atmospheric Stratification." *Boundary-Layer Meteorology* 191, no. 4 (April 2025): 19. https://doi.org/10.1007/s10546-025-00907-y.

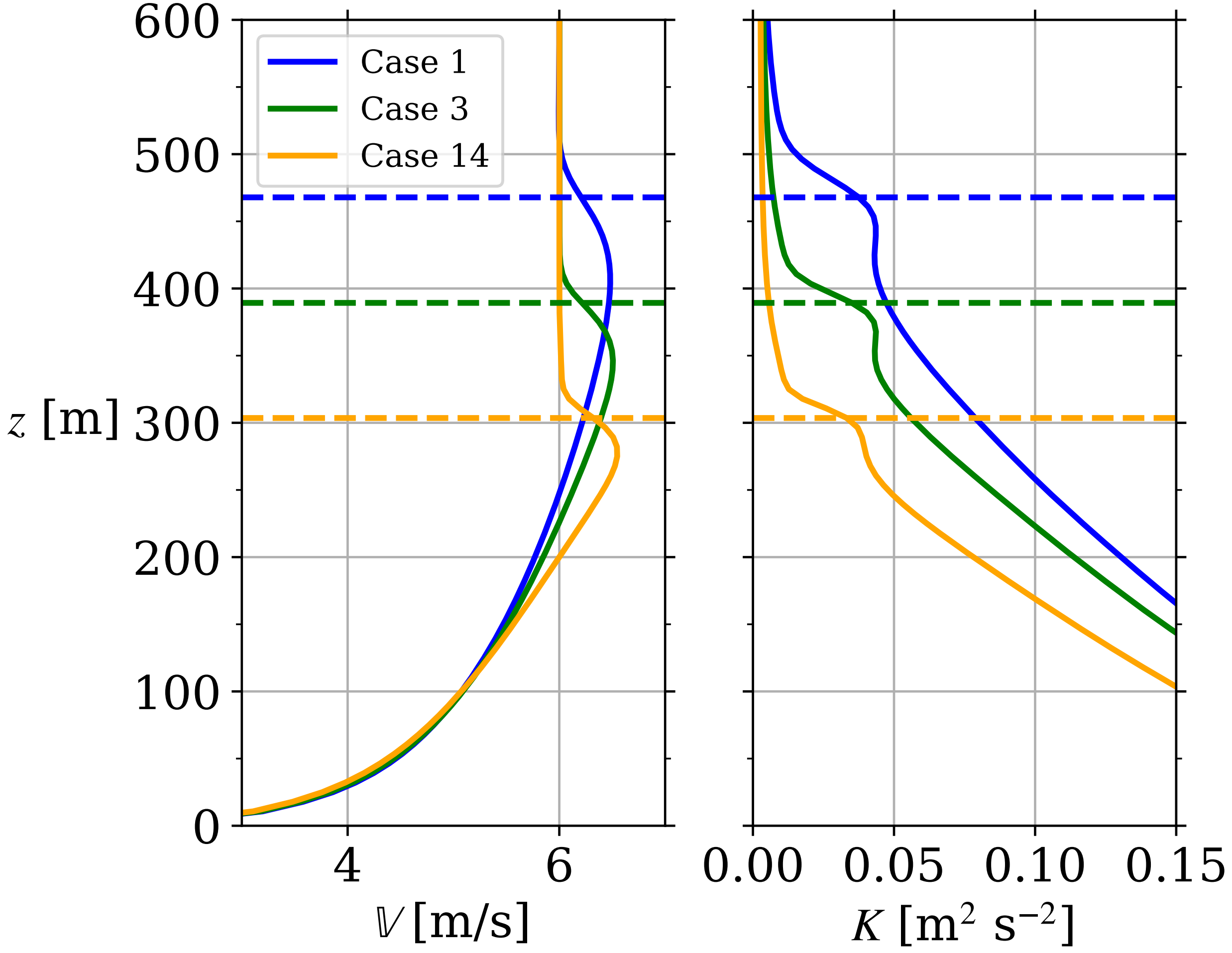
Ghobrial et al. (2025)

$$TI_u \equiv \frac{\sqrt{\overline{uu}}}{U_\infty}$$



[1]Liu, Luoqin, Xiyun Lu, and Richard J A M Stevens. “Geostrophic Drag Law in Conventionally Neutral Atmospheric Boundary Layer : Simplified Parametrization and Numerical Validation.” *Boundary-Layer Meteorology* 2 (2024). https://doi.org/10.1007/s10546-024-00878-6.

Liu et al. (2024)



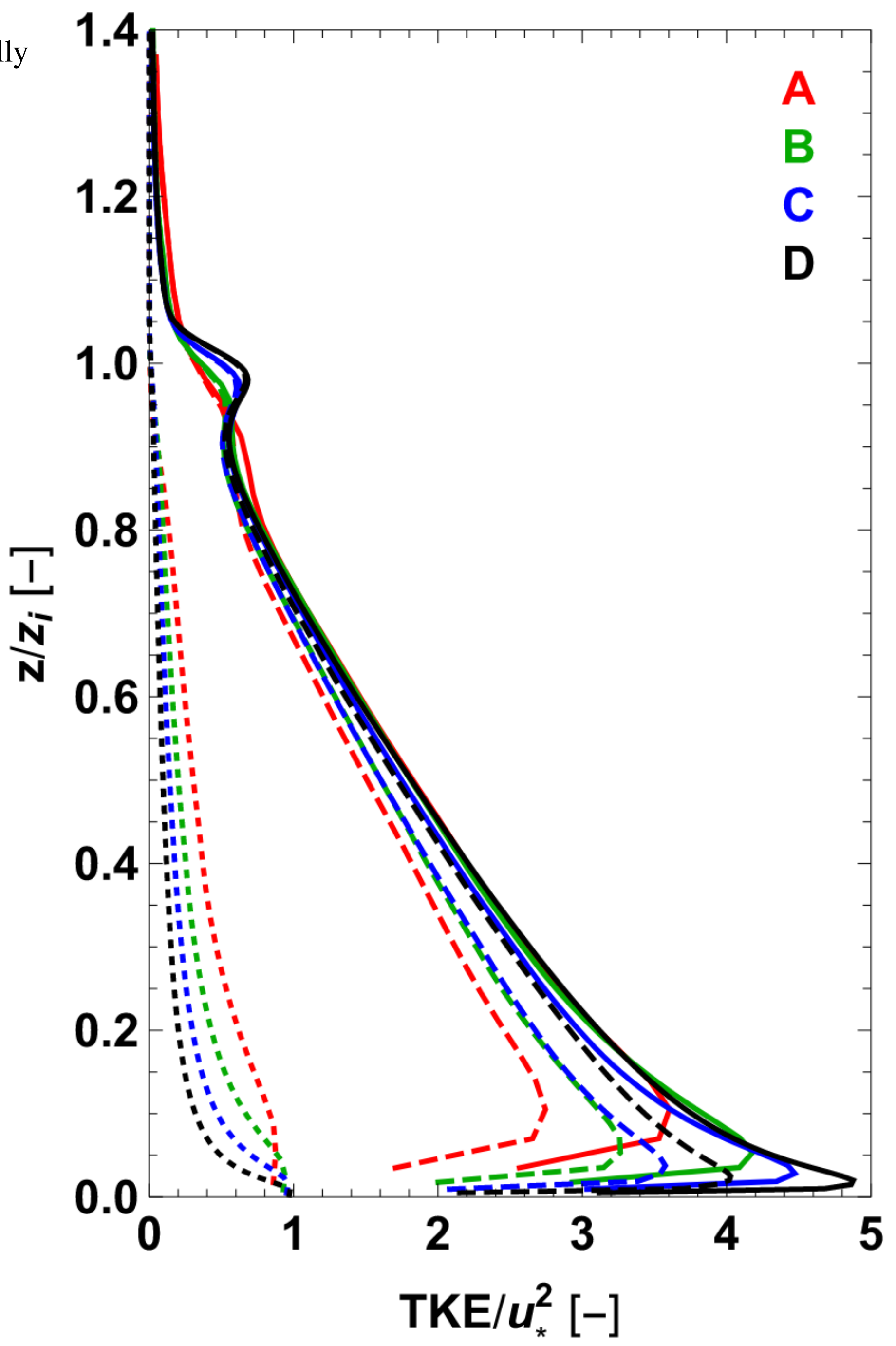
[1]Berg, Jacob, Edward G. Patton, and Peter P. Sullivan. "Large-Eddy Simulation of Conditionally Neutral Boundary Layers: A Mesh Resolution Sensitivity Study." *Journal of the Atmospheric Sciences*, no. 2010 (2020): 1969–91. https://doi.org/10.1175/jas-d-19-0252.1.

Berg et al. (2020)

Name	Grid points	(L_x, L_y, L_z) (m)
A	$128^2 \times 64$	(2560, 2560, 896)
B	$256^2 \times 128$	(2560, 2560, 896)
C	$512^2 \times 256$	(2560, 2560, 896)
D	$1024^2 \times 512$	(2560, 2560, 896)

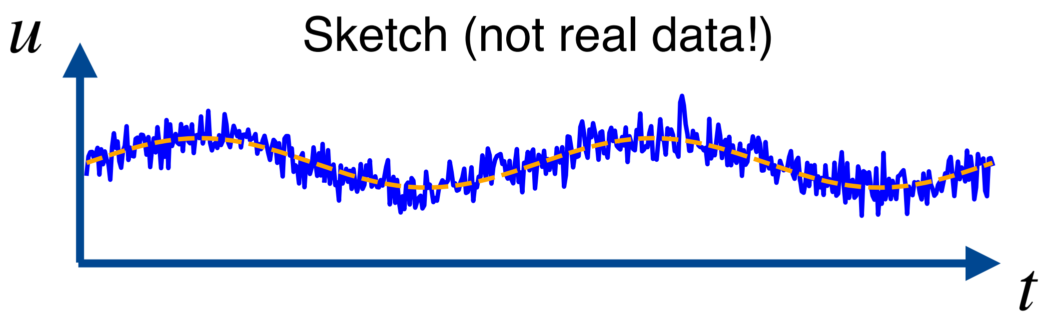
Name	h (m)	z_i (m)
A	348	399
B	353	395
C	338	372
D	321	350

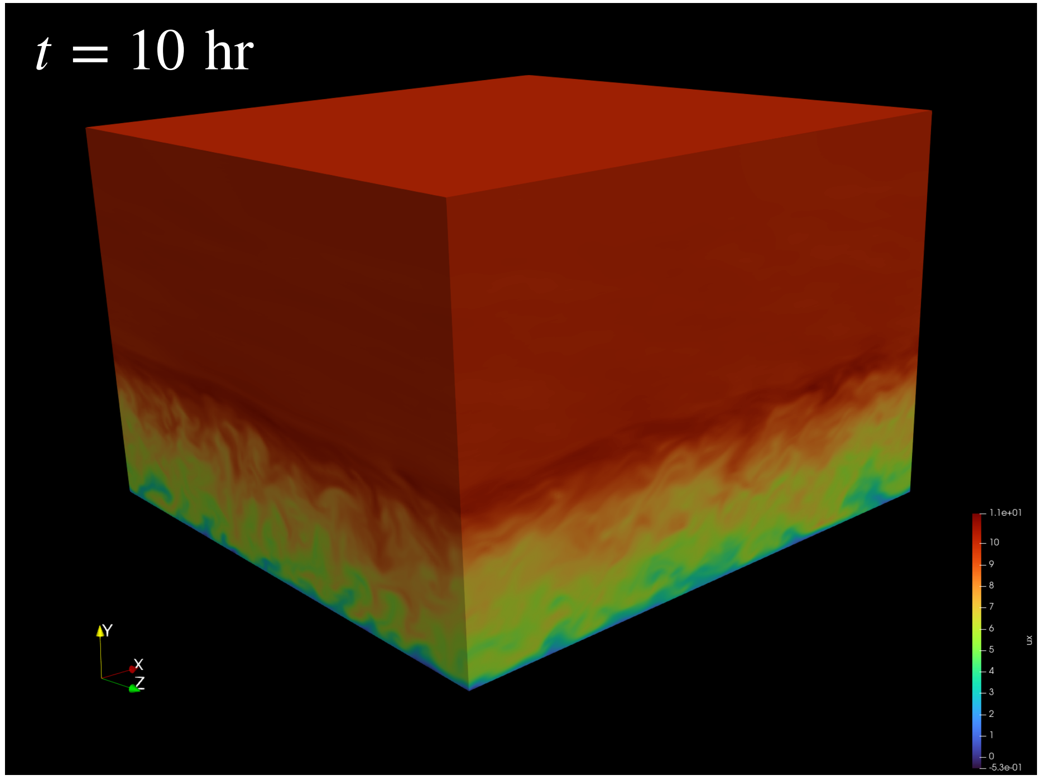
h: height where total momentum is 5% of surface value
z_i: height where the temperature gradient is maximum

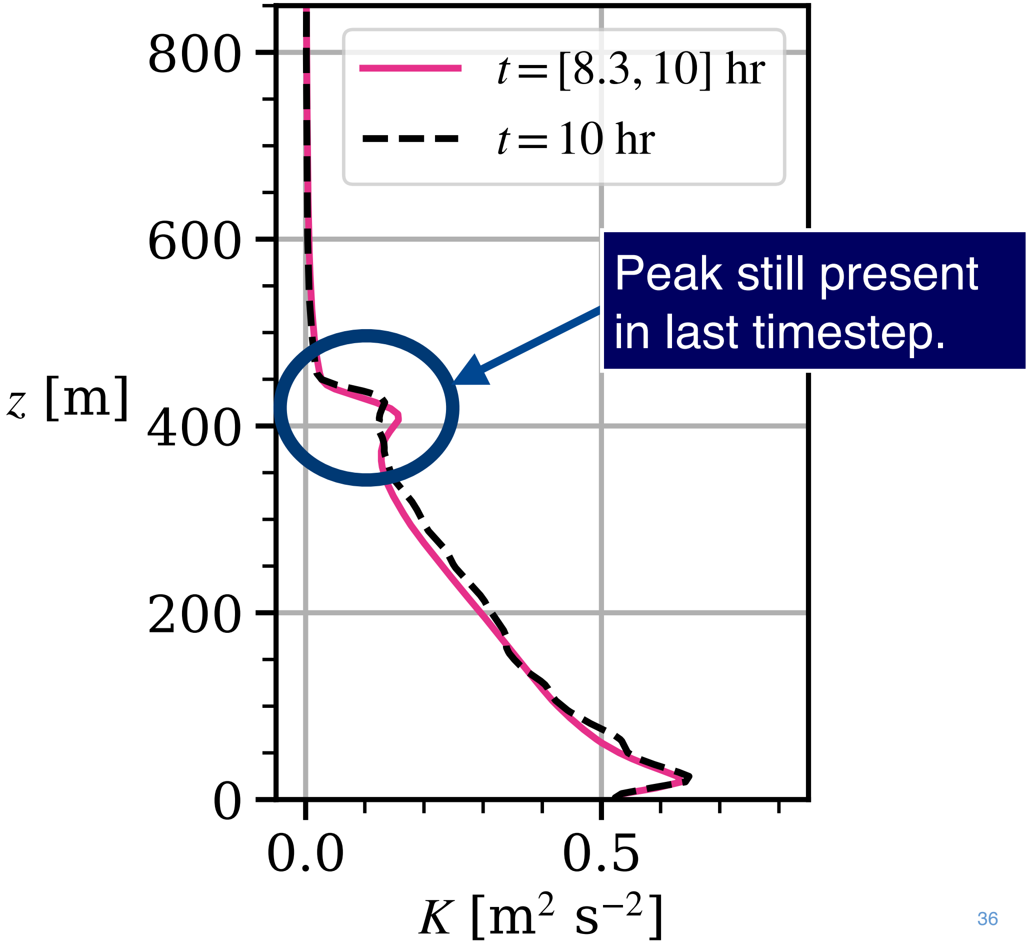


Non-turbulent oscillation?

- Time history of LES is not in the dataset.
- However, the snapshot of the last time step is.

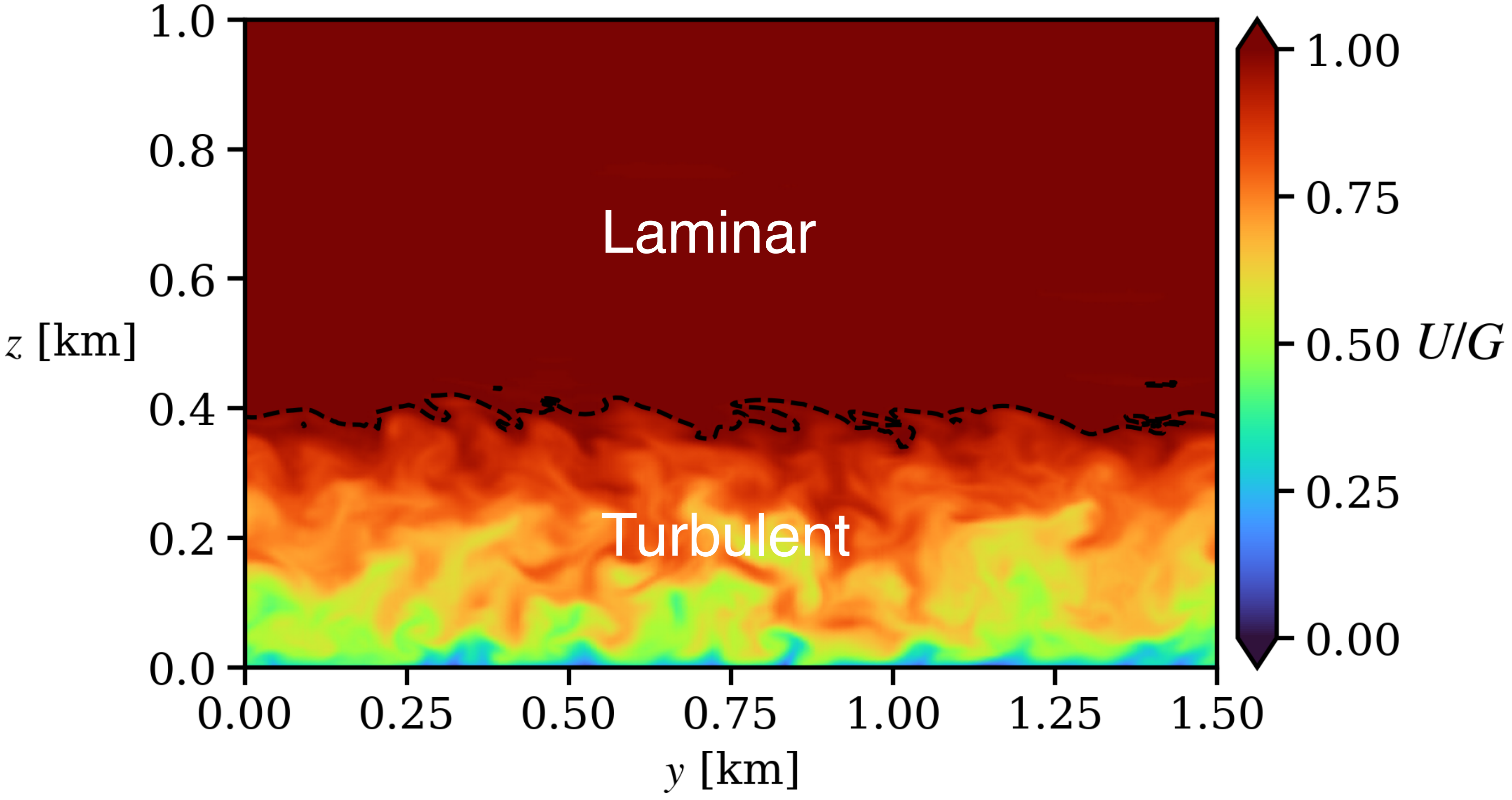






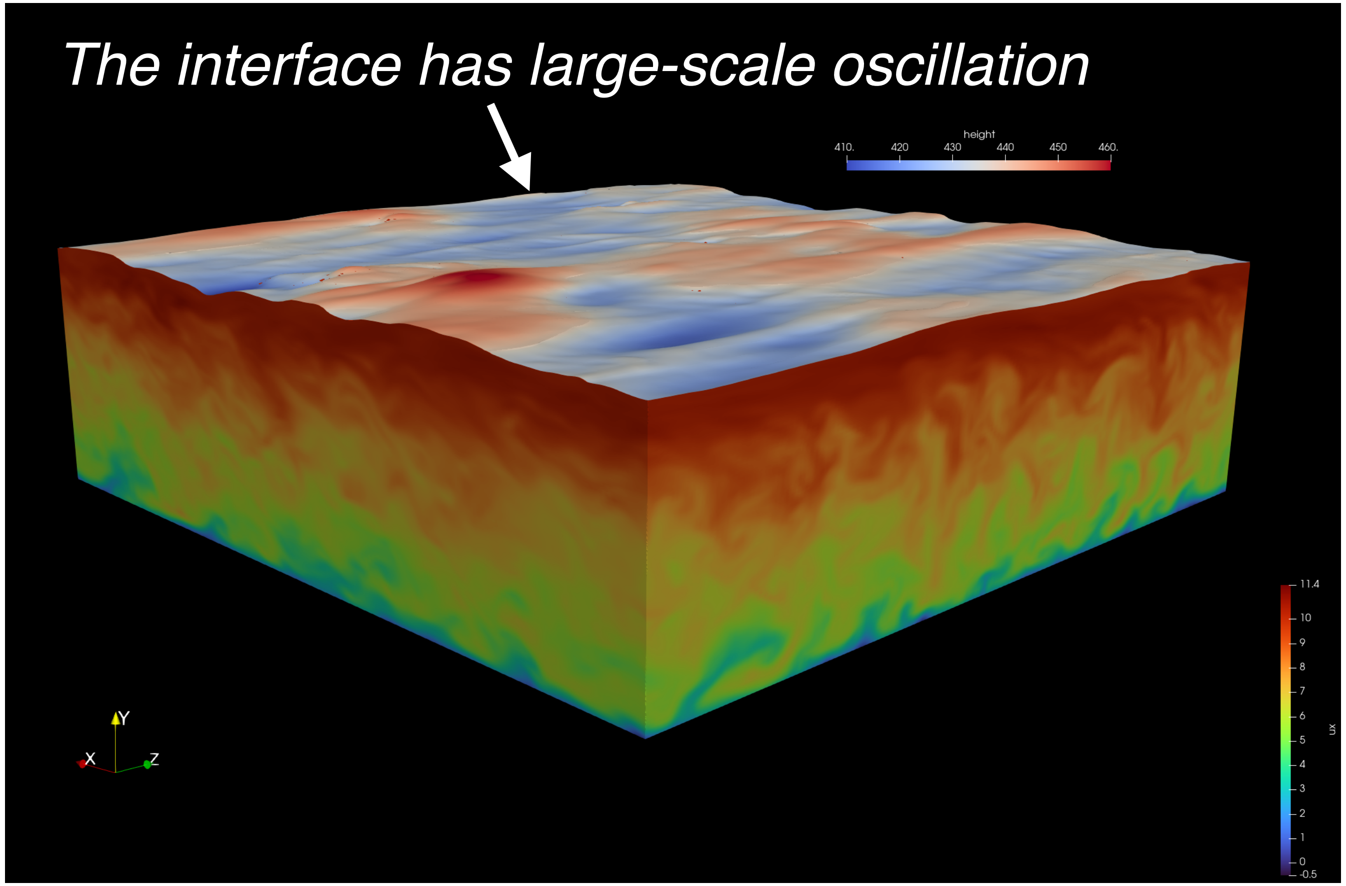
Non-turbulent oscillation in space?

- The ABL interface is thin and non-planar.



Non-turbulent oscillation in space?

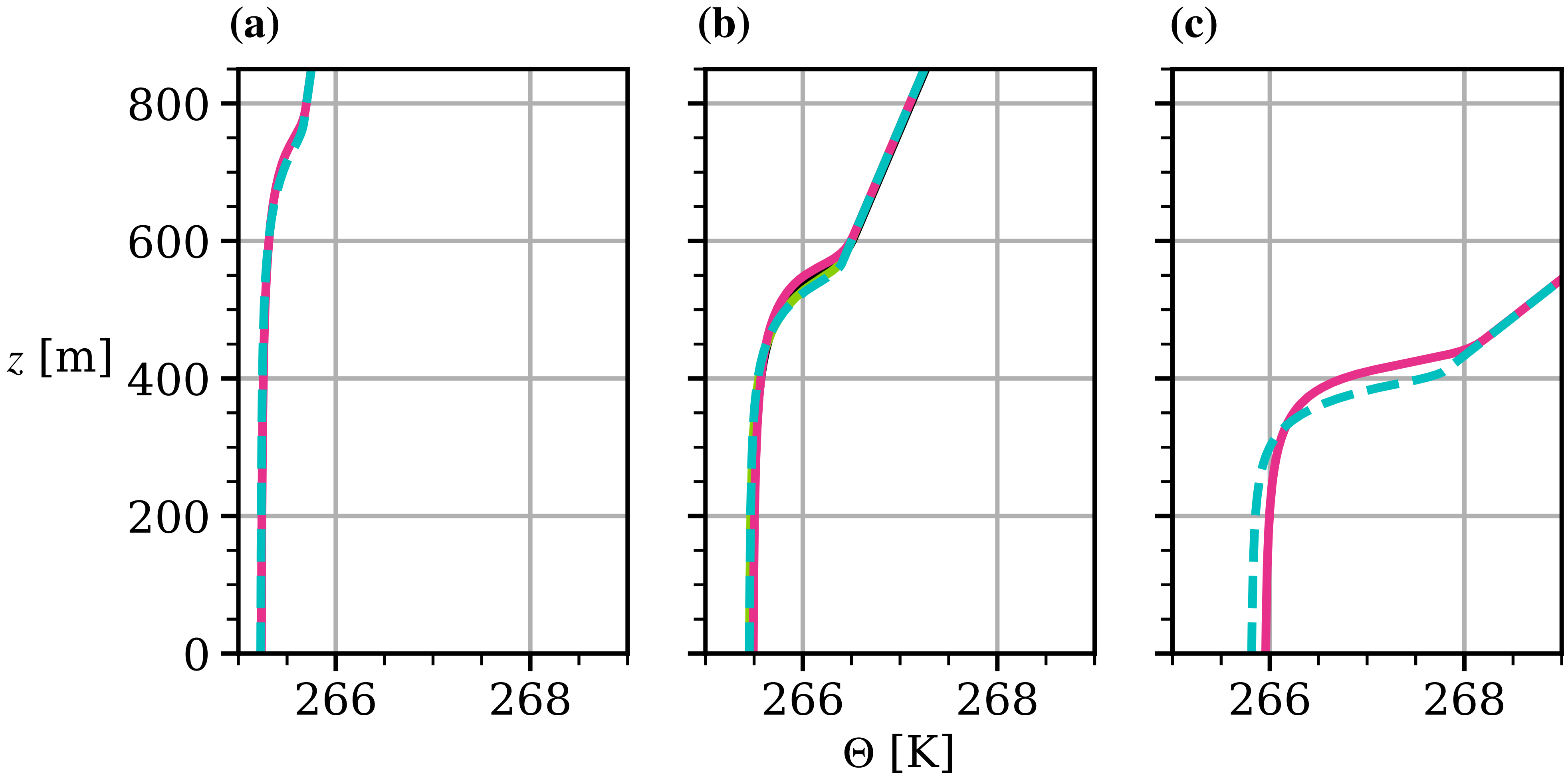
- 3D view of ABL interface (isocontour of temperature).



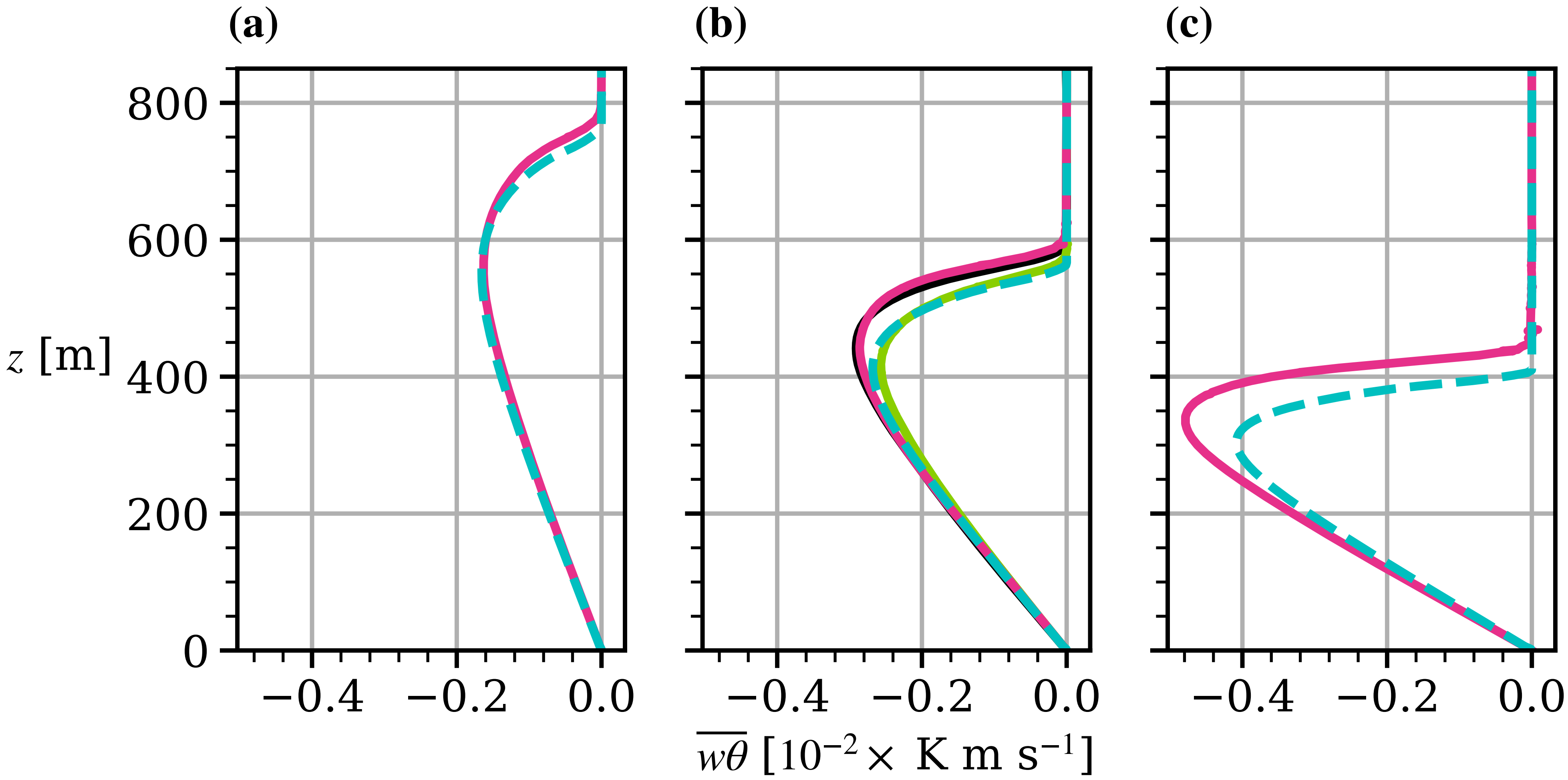
Maybe statistics should be taken over an undulating surface? Spectral analysis? Or maybe the peak is real?

Results (buoyancy)

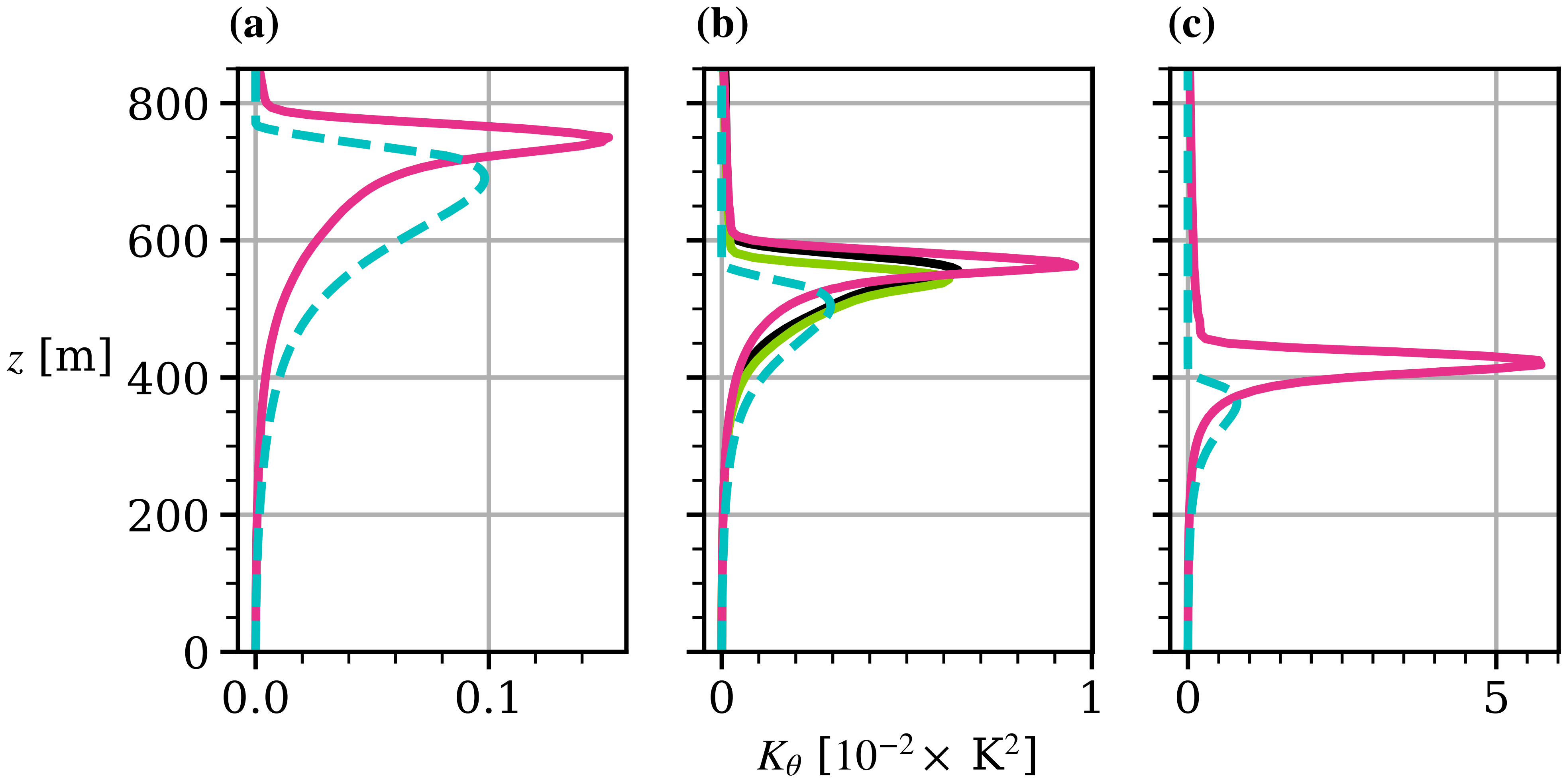
Temperature



Vertical heat flux



Half temperature variance

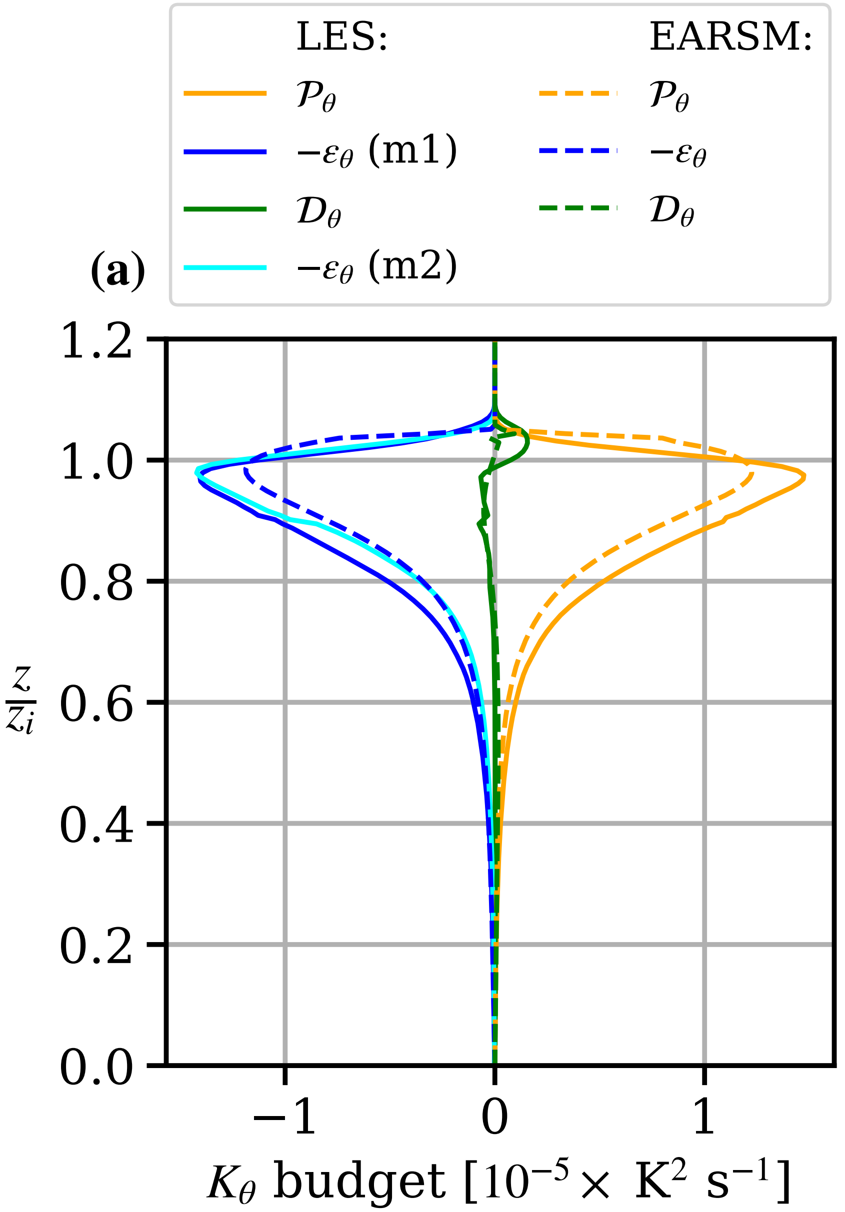


Half temperature variance budget

- For horizontally homogeneous flow:

$$\frac{\partial K_\theta}{\partial t} = -\overline{\theta' w'}\frac{\partial \Theta}{\partial z} - \kappa \overline{\frac{\partial \theta'}{\partial x_j}\frac{\partial \theta'}{\partial x_j}} - \frac{1}{2}\frac{\partial \overline{w'\theta'\theta'}}{\partial z}$$

- Data of $\overline{w'\theta'\theta'}$ is available in the NCAR data.
- Assume transient term is negligible and calculate dissipation as residual.
- Alternatively calculate as $\varepsilon_\theta = \dfrac{K_\theta \varepsilon}{rK}$.



Estimate of r from LES

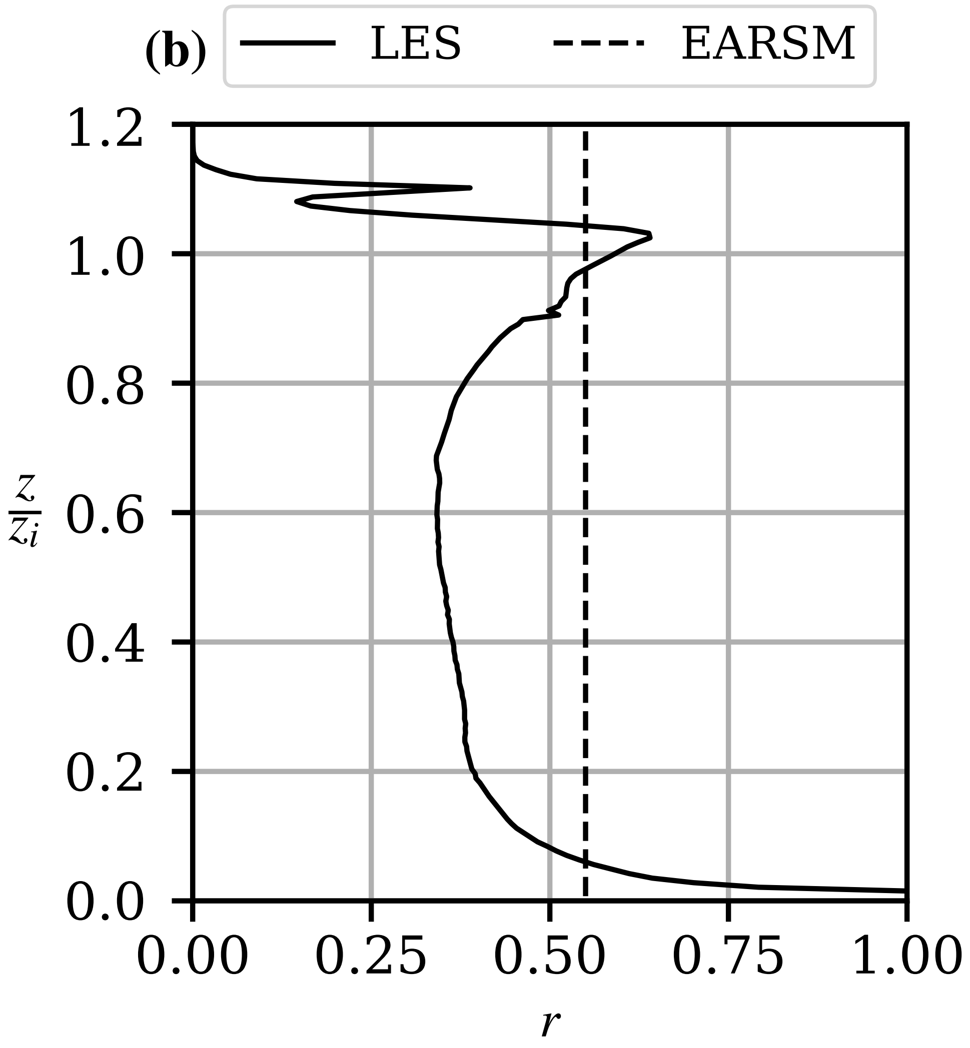
- In RANS, we model dissipation of K_θ as

$$\varepsilon_\theta = \frac{K_\theta \varepsilon}{rK}$$

- Re-arrange

$$r = \frac{K_\theta \varepsilon}{\varepsilon_\theta K}$$

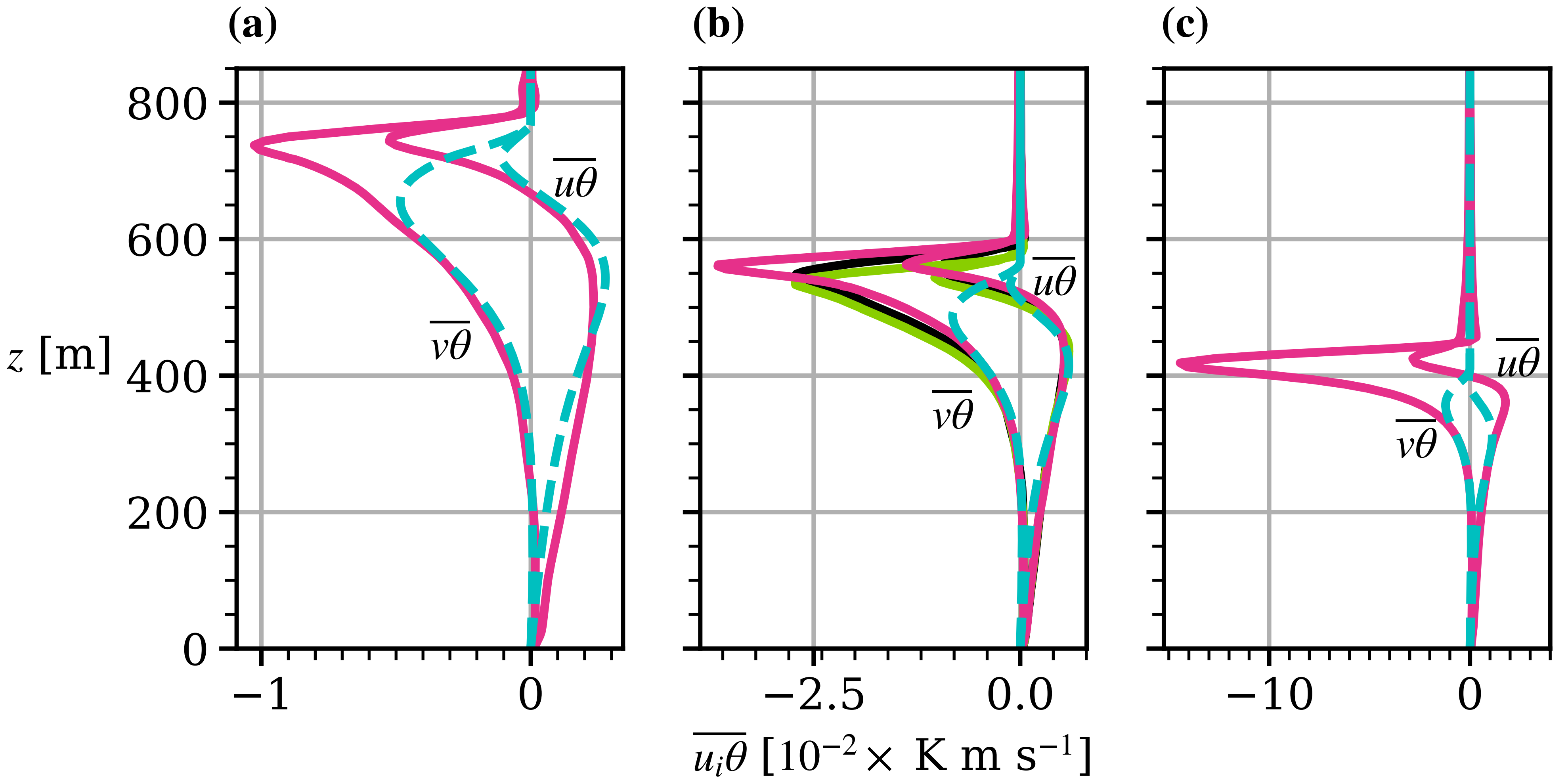
- Each term term on the RHS can be evaluated from LES.



EVMs have $\overline{u_i\theta} = -\kappa_t \frac{\partial \Theta}{\partial x_i}$,

hence no horizontal fluxes.

Horizontal heat fluxes



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[1]https://online.kitp.ucsb.edu/online/blayers-c18/sullivan/ at time 29m30s.
[2]Sullivan, Peter P., Jeffrey C. Weil, Edward G. Patton, Harmen J.J. Jonker, and Dmitrii V. Mironov. "Turbulent Winds and Temperature Fronts in Large-Eddy Simulations of the Stable Atmospheric Boundary Layer." *Journal of the Atmospheric Sciences* 73, no. 4 (April 1, 2016): 1815–40. https://doi.org/10.1175/JAS-D-15-0339.1.

Sullivan's explanation of horizontal heat fluxes[1]

- Horizontal fluxes also appear in the SBL[2].
- For horizontally homogeneous flow:

$$\frac{\partial \overline{u\theta}}{\partial t} = -\overline{w\theta}\frac{\partial U}{\partial z} - \overline{uw}\frac{\partial \Theta}{\partial z} - \frac{\partial \overline{uw\theta}}{\partial z} - \frac{1}{\rho}\overline{\theta\frac{\partial p}{\partial x}}$$

- Say the transient, buoyant and turbulence transport terms are small.

$$0 \sim -\overline{w\theta}\frac{\partial U}{\partial z} - \frac{1}{\rho}\overline{\theta\frac{\partial p}{\partial x}}$$

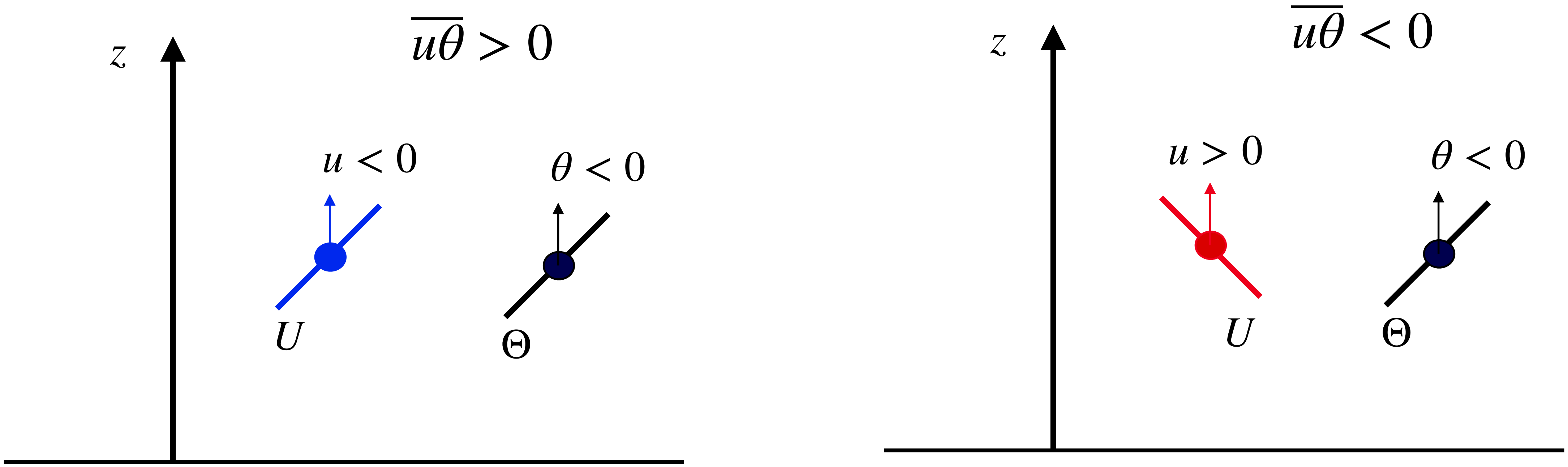
- Use Rotta model for the pressure transport term

$$\frac{1}{\rho}\overline{\theta\frac{\partial p}{\partial x}} \sim \frac{\overline{u\theta}}{\tau}$$

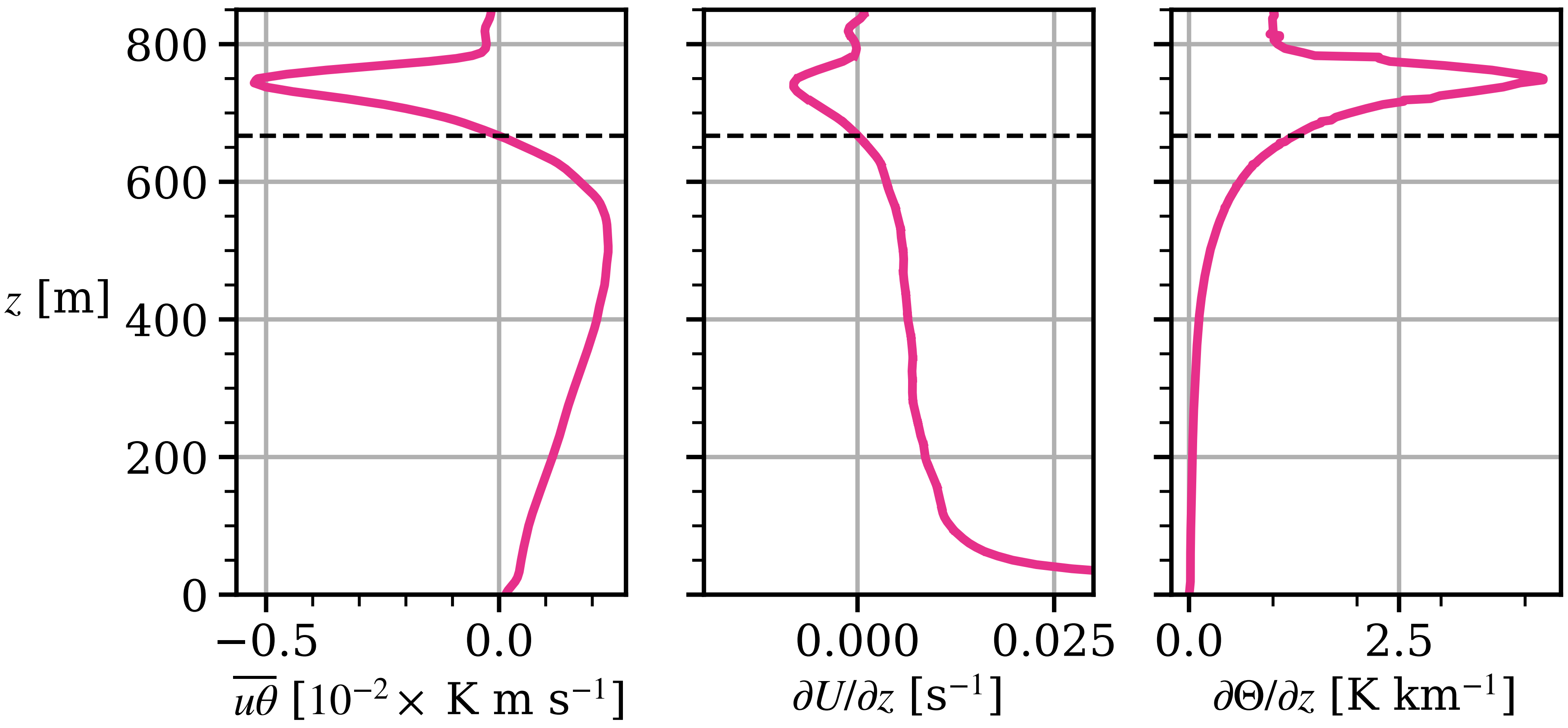
$$\boxed{\overline{u\theta} \sim \tau\overline{w\theta}\frac{\partial U}{\partial z}}$$

A simpler explanation of horizontal heat fluxes

- Consider an upward perturbation of a particle



A simpler explanation of horizontal heat fluxes



Conclusions and future work

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Conclusions

1. Buoyant EARSM has good predictions for most variables in CNBLs. It is $\mathcal{O}(10^6)$ faster than LES.
2. LES ABL can behave as low-Re fluid near the wall → be careful with comparisons!

Future work

1. Better understanding of K and K_θ peaks in inversion layer
2. OpenFOAM implementation of buoyant EARSM
3. A simplified constitutive relation