Validation and extension of an analytic momentum availability model for the two-scale momentum theory of wind farm flows

Mads Baungaard[1], Takafumi Nishino[1] and Andrew Kirby[2]

[1]University of Oxford

[2]Coal Lane, London

Group meeting, 4 March 2026

UNIVERSITY OF OXFORD

arXiv link: https://arxiv.org/abs/2602.10126

Recent preprint

arXiv > physics > arXiv:2602.10126

Search...
Help | Ad

Physics > Fluid Dynamics

[Submitted on 2 Feb 2026]

Validation and extension of an analytic momentum availability model for the two-scale momentum theory of wind farm flows

Mads Baungaard, Takafumi Nishino, Andrew Kirby

A key parameter in the two-scale momentum theory of wind farm flows is the momentum availability, which quantifies the supply of momentum to a wind farm from various different momentum transport mechanisms (advection, pressure gradient, Coriolis, turbulence and unsteadiness). In this study, the contribution of each of these mechanisms to the momentum availability is evaluated directly from large-eddy simulation (LES) data in order to validate an analytic momentum availability model (Kirby, Dunstan, & Nishino, J. Fluid Mech., vol. 976, 2023, A24). Application of the model to six wind farm cases, three with different atmospheric boundary-layer (ABL) heights and three with different turbine layouts, shows that the full model performs well across all cases, but that its linearized version increasingly overpredicts the momentum availability for increasing ABL heights. It is found that the overprediction is related to the ABL Rossby number, and based on this observation, we propose an extension of the original linear model, which improves its accuracy for the considered cases and makes it more generally applicable, in particular to cases with tall ABL heights or strong Coriolis forcing.

Subjects: **Fluid Dynamics (physics.flu-dyn)**
Cite as: arXiv:2602.10126 **[physics.flu-dyn]**
(or arXiv:2602.10126v1 **[physics.flu-dyn]** for this version)
https://doi.org/10.48550/arXiv.2602.10126

Submission history

From: Mads Baungaard [view email]
[v1] Mon, 2 Feb 2026 10:58:18 UTC (3,346 KB)

Content

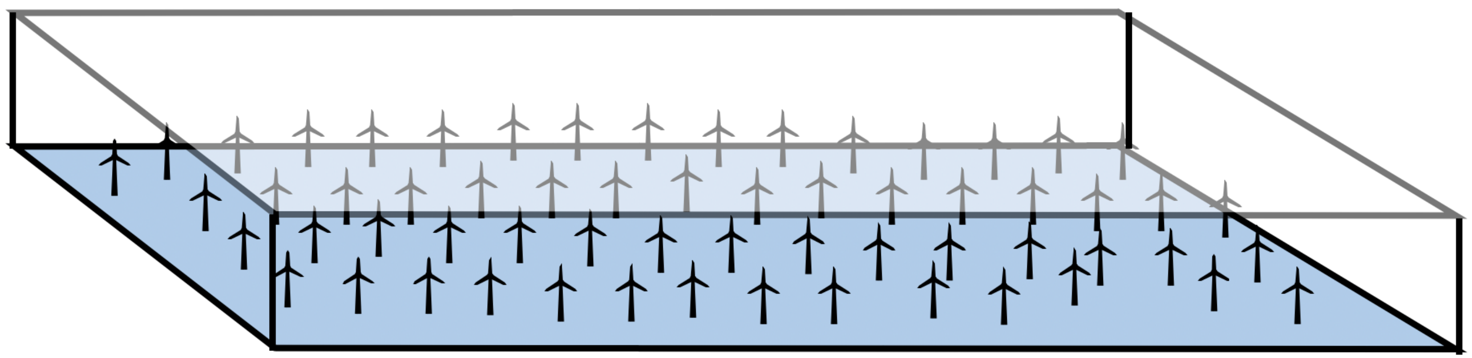
1. Introduction to the two-scale momentum theory.
2. Momentum availability from LES data.
3. ABL Rossby extension of an analytic momentum availability model.

1. Introduction to the two-scale momentum theory

Nishino, T., and T. D. Dunstan. "Two-Scale Momentum Theory for Time-Dependent Modelling of Large Wind Farms." *Journal of Fluid Mechanics* 894 (2020): A2. https://doi.org/10.1017/jfm.2020.252.

What is the two-scale momentum theory?

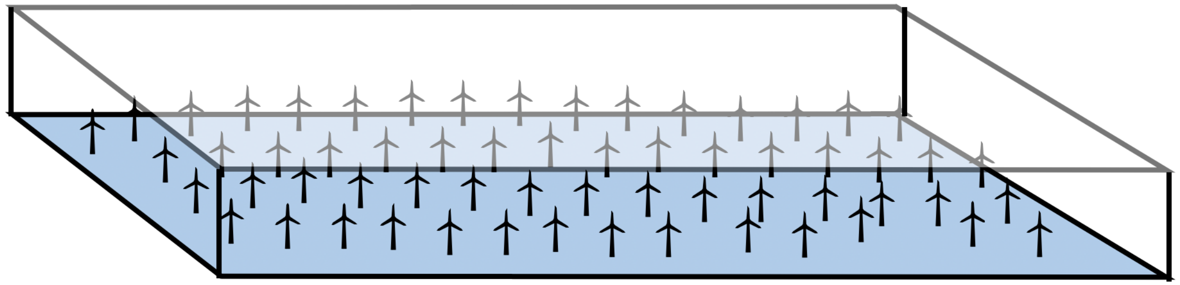
Control volume (CV) analysis: $\underbrace{\text{Turbine drag} + \text{Ground friction}}_{\text{momentum sinks}} = \text{Momentum supply to CV}$



With this simple idea one can predict **wind farm efficiency**!

Wind farm efficiency

During one hour, this wind farm delivered 148.3 MW on average.



Q:*Is this a lot or not?*

A: Depends on the number of turbines N_t, rotor diameter D, air density ρ and wind resource. It is better to ask, "what is its efficiency?".

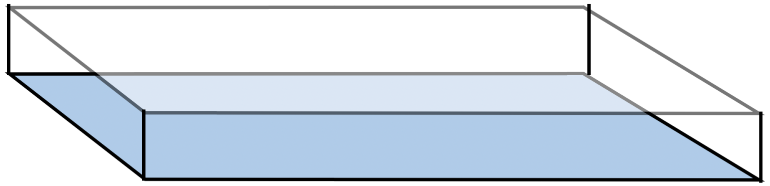
Global power coefficient:

$$C_{PG} = \sum_{i=1}^{N} P_i / \left(\frac{1}{2} \rho N_t A_d U_{F0}^3 \right)$$

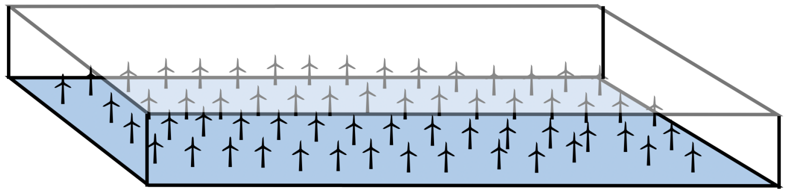
Farm speed

The streamwise velocity volume-averaged over the CV

without turbines, U_{F0}



and with turbines, U_F.



An important variable in the theory is the wind-speed reduction factor

$$\beta \equiv \frac{U_F}{U_{F0}},$$

Obviously, $0 < \beta < 1$.

UNIVERSITY OF OXFORD

The non-dimensional farm momentum (NDFM) equation

The most important equation in the two-scale momentum theory:

$$\underbrace{C_T^* \frac{\lambda}{C_{f0}} \beta^2}_{\text{Normalized turbine drag}} + \underbrace{\beta^\gamma}_{\text{Normalized wall friction}} = \underbrace{M}_{\text{Momemtum availability}}$$

Parameters: λ (array density), C_{f0} (surface friction coefficient) and γ (friction exponent).
Models: C_T^* (internal thrust coefficient) and M (momentum availability).
Output: β (wind-speed reduction factor).

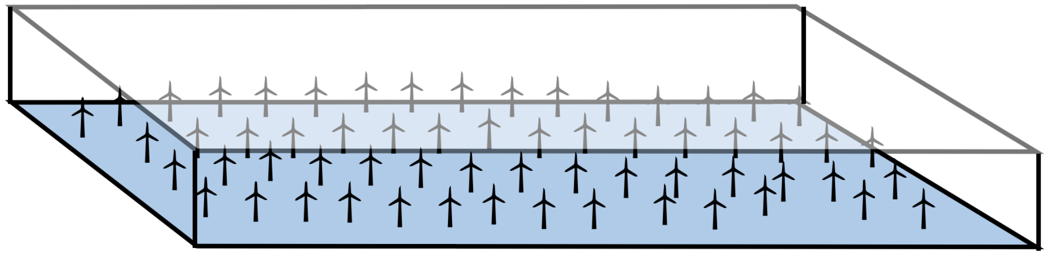
UNIVERSITY OF OXFORD

Derivation of NDFM equation (1/3)

Start from U-mom'm equation of RANS

$$\frac{\partial \rho U_1}{\partial t} = -\frac{\partial \rho U_1 U_j}{\partial x_j} - \frac{\partial P}{\partial x_1} + \rho f_c U_2 + \frac{\partial \tau_{1j}}{\partial x_j} + f_1 \quad (\text{unit}: \ \text{N m}^{-3})$$

Volume-average over CV (square brackets) and use Gauss integral theorem on advection and stress terms



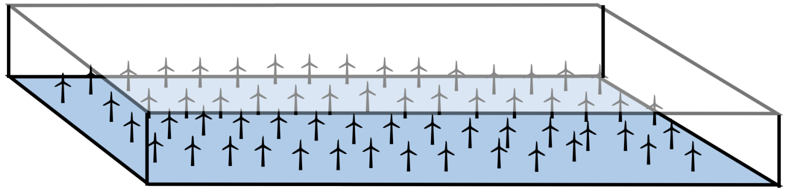
$$\left[\frac{\partial \rho U_1}{\partial t}\right] = -\frac{1}{V_{\text{cv}}}\int_{\Omega_{\text{cv}}} \rho U_1 U_j dA_j - \left[\frac{\partial P}{\partial x_1} - \rho f_c U_2\right] - \frac{S_{\text{cv}}\tau_w}{V_{\text{cv}}} + \frac{1}{V_{\text{cv}}}\int_{\Omega_{\text{cv}}\backslash\Omega_w} \tau_{1j} dA_j - \frac{T}{V_{\text{cv}}} \quad (\text{unit}: \ \text{N m}^{-3})$$

Derivation of NDFM equation (2/3)

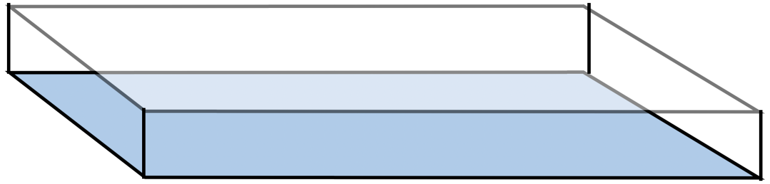
Multiply through with V_{cv} and re-arrange

$$T + S_{\mathrm{cv}}\tau_w = \underbrace{-\int_{\Omega_{\mathrm{cv}}} \rho U_1 U_j dA_j}_{X_{\mathrm{adv}}} \underbrace{-V_{\mathrm{cv}}\left[\frac{\partial P}{\partial x_1}\right]}_{X_{\mathrm{PGF}}} + \underbrace{V_{\mathrm{cv}}\left[\rho f_c U_2\right]}_{X_{\mathrm{Cor}}} + \underbrace{\int_{\Omega_{\mathrm{cv}}\setminus\Omega_w} \tau_{1j} dA_j}_{X_{\mathrm{turb}}} \underbrace{-V_{\mathrm{cv}}\left[\frac{\partial \rho U_1}{\partial t}\right]}_{X_{\mathrm{uns}}} \quad (\mathrm{unit:}\ \ \mathrm{N})$$

With the shorthand 'X'-notation:



$$T + S_{\mathrm{cv}}\tau_w = X_{\mathrm{adv}} + X_{\mathrm{PGF}} + X_{\mathrm{Cor}} + X_{\mathrm{turb}} + X_{\mathrm{uns}} \quad (\mathrm{unit:}\ \ \mathrm{N})$$



$$S_{\mathrm{cv}}\tau_{w0} = X_{\mathrm{adv0}} + X_{\mathrm{PGF0}} + X_{\mathrm{Cor0}} + X_{\mathrm{turb0}} + X_{\mathrm{uns0}} \quad (\mathrm{unit:}\ \ \mathrm{N})$$

Derivation of NDFM equation (3/3)

Definitions for LHS:

$\gamma \equiv \log_{\beta}\left(\tau_w/\tau_{w0}\right)$ $\quad C_{f0} \equiv \tau_{w0}/\left(\frac{1}{2}\rho U_{F0}^2\right)$

$C_T^* \equiv T/\left(\frac{1}{2}\rho U_F^2 N_t A_d\right)$ $\quad \beta \equiv U_F/U_{F0}$

$\lambda \equiv N_t A_d/S_{cv}$ $\quad U_F \equiv [U_1]$

Use the ‘0’-equation to normalize

$$\underbrace{\frac{T}{S_{\mathrm{cv}}\tau_{w0}}}_{C_T^*\frac{\lambda}{C_{f0}}\beta^2} + \underbrace{\frac{S_{\mathrm{cv}}\tau_w}{S_{\mathrm{cv}}\tau_{w0}}}_{\beta^\gamma} = \underbrace{\frac{X_{\mathrm{adv}} + X_{\mathrm{PGF}} + X_{\mathrm{Cor}} + X_{\mathrm{turb}} + X_{\mathrm{uns}}}{X_{\mathrm{adv0}} + X_{\mathrm{PGF0}} + X_{\mathrm{Cor0}} + X_{\mathrm{turb0}} + X_{\mathrm{uns0}}}}_{M} \quad (\mathrm{unit:}\ -)$$

This is the NDFM equation:

$$C_T^*\frac{\lambda}{C_{f0}}\beta^2 + \beta^\gamma = M \quad (\mathrm{unit:}\ -)$$

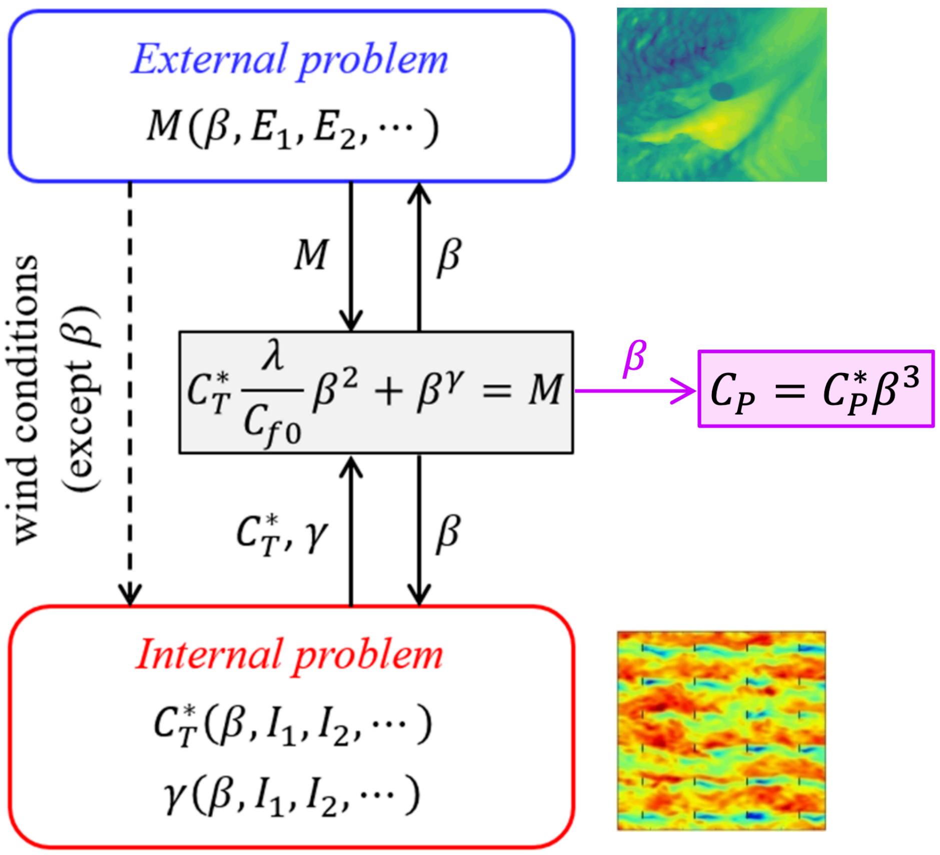
Why is it called *two-scale* momentum theory?

$$\underbrace{C_T^* \frac{\lambda}{C_{f0}} \beta^2}_{\text{Turbine drag}} + \underbrace{\beta^\gamma}_{\text{Wall friction}} = \underbrace{M}_{\text{Momemtum availability}}$$

Scale 1: C_T^* and γ mainly depend on turbine/array-scale flow physics.

Scale 2: M mainly depends on farm/atmospheric-scale flow physics.

Scale 1+2: β depends on both.



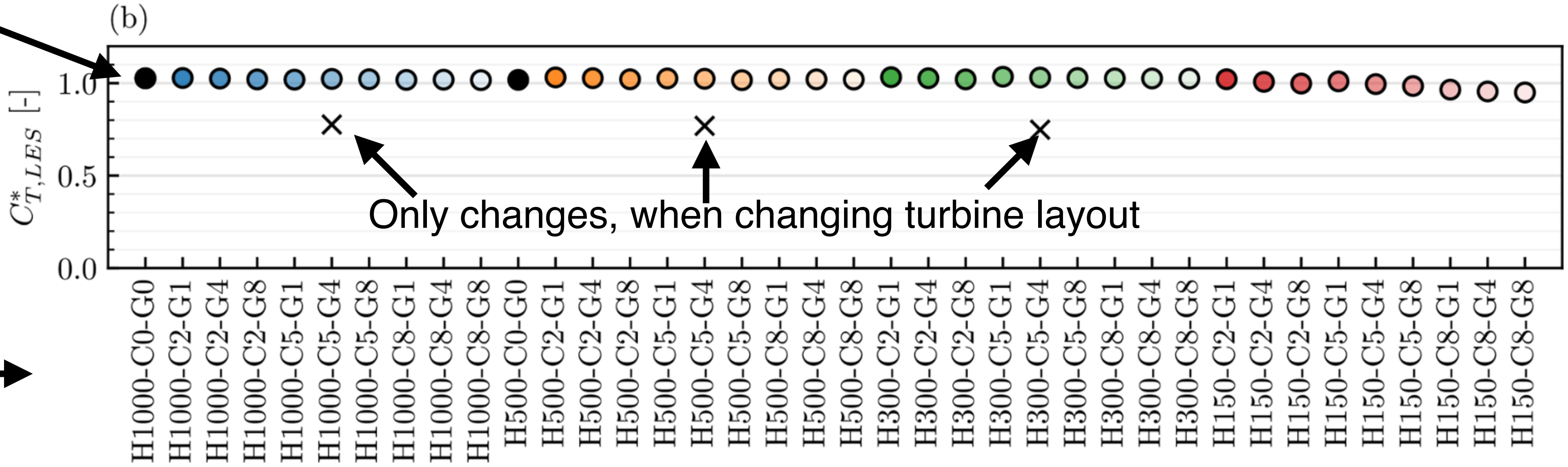
Nishino, T., and T. D. Dunstan. "Two-Scale Momentum Theory for Time-Dependent Modelling of Large Wind Farms." *Journal of Fluid Mechanics* 894 (2020): A2. https://doi.org/10.1017/jfm.2020.252.

Kirby, A., T. Nishino, L. Lanzilao, T. D. Dunstan, and J. Meyers. "Turbine- and Farm-Scale Power Losses in Wind Farms: An Alternative to Wake and Farm Blockage Losses." *Wind Energy Science* 10, no. 2 (2025): 435–50. https://doi.org/10.5194/wes-10-435-2025.

Evidence of two-scale separation assumption

C_T^* does **not** depend on the ABL

A lot of different ABLs



Nishino, T., and T. D. Dunstan. "Two-Scale Momentum Theory for Time-Dependent Modelling of Large Wind Farms." *Journal of Fluid Mechanics* 894 (2020): A2. https://doi.org/10.1017/jfm.2020.252.

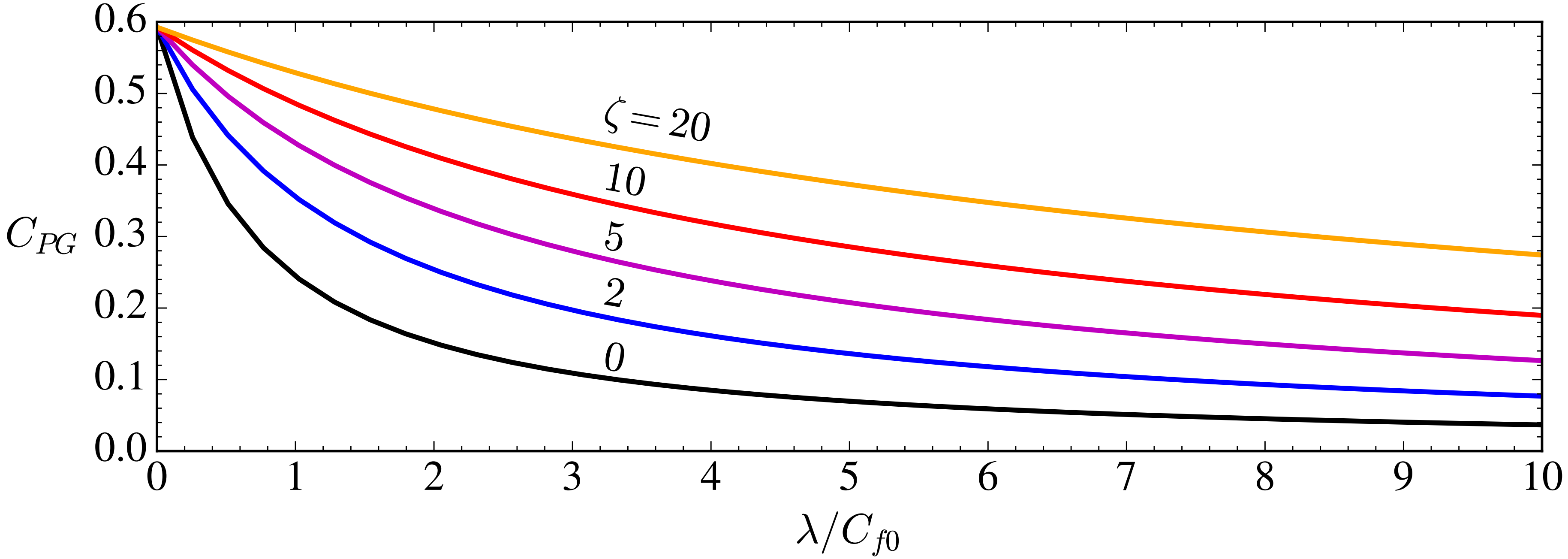
Analytical prediction of C_{PG} with two-scale momentum theory

Step 1: Solve for β
(requires model of C_T^*, γ and M)

$$C_T^* \frac{\lambda}{C_{f0}} \beta^2 + \beta^\gamma = M$$

Step 2: Obtain C_{PG}
(requires model for C_P^*)

$$C_{PG} = \beta^3 C_P^*$$



Required models for the two-scale momentum theory

Four models are needed:

- Internal thrust coefficient, C_T^*.
- Internal power coefficient, C_P^*.
- Friction exponent, γ.
- Momentum availability, M.

See Kirby et al. (JFM2022) and Kirby et al. (WE2023).

This presentation is about M.

2. Momentum availability from LES data

Momentum availability definition

- The mathematical definition is

$$M = \frac{X_F}{X_{F0}} = \frac{X_{\text{adv}} + X_{\text{PGF}} + X_{\text{Cor}} + X_{\text{turb}} + X_{\text{uns}}}{X_{\text{adv0}} + X_{\text{PGF0}} + X_{\text{Cor0}} + X_{\text{turb0}} + X_{\text{uns0}}}$$

M represents <u>*how much the "available" momentum differs from its "natural" value.*</u>

If there is no farm-ABL response, $M = 1$.

ΔM-contributions

- Can re-write M to

$$M = \frac{X_F}{X_{F0}} = 1 + \frac{X_F - X_{F0}}{X_{F0}} = 1 + \underbrace{\frac{X_{\text{adv}} - X_{\text{adv0}}}{X_{F0}}}_{\Delta M_{\text{adv}}} + \underbrace{\frac{X_{\text{PGF}} - X_{\text{PGF0}}}{X_{F0}}}_{\Delta M_{\text{PGF}}} + \underbrace{\frac{X_{\text{Cor}} - X_{\text{Cor0}}}{X_{F0}}}_{\Delta M_{\text{Cor}}} + \underbrace{\frac{X_{\text{turb}} - X_{\text{turb0}}}{X_{F0}}}_{\Delta M_{\text{turb}}} + \underbrace{\frac{X_{\text{uns}} - X_{\text{uns0}}}{X_{F0}}}_{\Delta M_{\text{uns}}}$$

$$\boxed{M = 1 + \Delta M_{\text{adv}} + \Delta M_{\text{PGF}} + \Delta M_{\text{Cor}} + \Delta M_{\text{turb}} + \Delta M_{\text{uns}}}$$

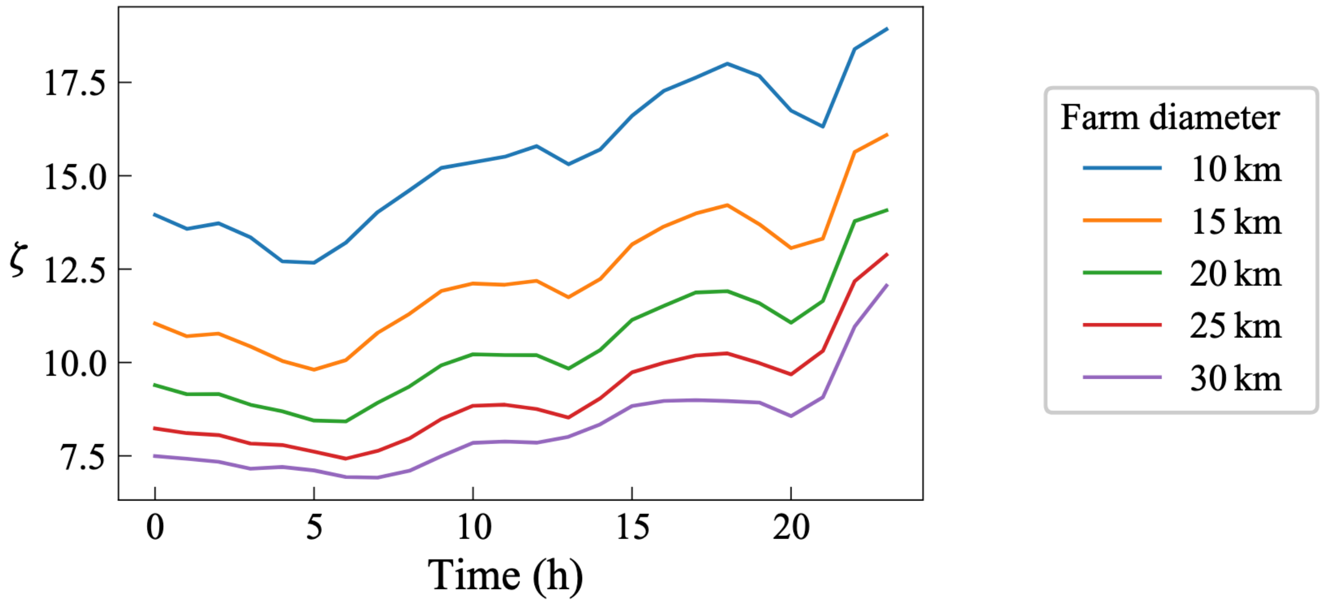
Patel, K., T. D. Dunstan, and T. Nishino. "Time-Dependent Upper Limits to the Performance of Large Wind Farms Due to Mesoscale Atmospheric Response." *Energies* 14, no. 19 (2021): 6437. https://doi.org/10.3390/en14196437.
Kirby, A., T. Nishino, and T. D. Dunstan. "Two-Scale Interaction of Wake and Blockage Effects in Large Wind Farms." *Journal of Fluid Mechanics* 953 (2022): A39. https://doi.org/10.1017/jfm.2022.979.

A linear model of M

- Nishino & Dunstan (2020) introduced a simple linear model:

$$M = 1 + \zeta(1 - \beta)$$

- It has one parameter, ζ ("wind response factor" or "extractability factor"). Patel et al. (2021) and Kirby et al. (2022) indirectly estimated this from NWP simulations.



Kirby, Andrew, Thomas D. Dunstan, and Takafumi Nishino. "An Analytical Model of Momentum Availability for Predicting Large Wind Farm Power." *Journal of Fluid Mechanics* 976 (2023): 1–22. https://doi.org/10.1017/jfm.2023.844.

M_{KDN1}-model

Sub-models:

$$\Delta M_{\mathrm{adv}} = \frac{H_F}{LC_{f0}} \left(\beta_{\mathrm{local}}(0)^2 - \beta_{\mathrm{local}}(L)^2\right)$$

$$\Delta M_{\mathrm{PGF}} = 2\frac{H_F}{LC_{f0}} \left(1 - \beta_{\mathrm{local}}(0)^2\right)$$

$$\Delta M_{\mathrm{Cor}} = 0$$

$$\Delta M_{\mathrm{turb}} = M + M\left(\frac{\tau_{t0}}{\tau_{w0}} - 1\right)\frac{h_0}{h} - \frac{\tau_{t0}}{\tau_{w0}}$$

$$\Delta M_{\mathrm{uns}} = 0$$

Advection-pressure (AP) approximation:

$$\Delta M_{\mathrm{adv}} + \Delta M_{\mathrm{PGF}} = \frac{H_F}{LC_{f0}} \left(1 - \beta^2\right)$$

Boundary layer height approximation:

$$\frac{h_0}{h} = \beta$$

Combine to get

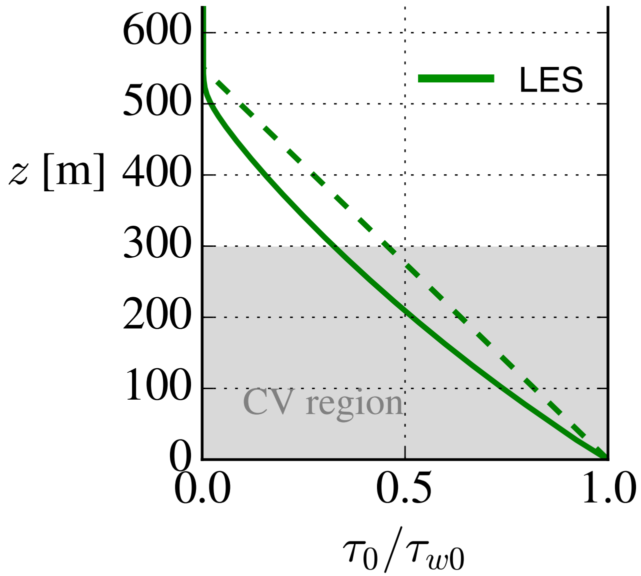
$$M_{\mathrm{KDN1}} = \frac{1 + \frac{H_F}{LC_{f0}}\left(1 - \beta^2\right) - \frac{\tau_{t0}}{\tau_{w0}}}{\beta\left(1 - \frac{\tau_{t0}}{\tau_{w0}}\right)}$$

M_{KDN2}-model

- A simplified version of the M_{KDN1} model.
- Idea is to model τ_{t0}/τ_{w0} by assuming a linear $\tau_0(z)$-profile.

Require $\tau_0(z)$ to be linear all the way up to the ABL height, h_0.

Linear $\tau_0(z)$-profile: $\tau_0(z) = \tau_{w0} + \dfrac{0 - \tau_{w0}}{h_0} z \quad , \quad 0 < z < h_0$ $\Rightarrow$ **Linearization:** $1 - \dfrac{\tau_{t0}}{\tau_{w0}} \approx \dfrac{H_F}{h_0}$



$$M_{\text{KDN2}} = \frac{1 + \frac{h_0}{LC_{f0}}\left(1 - \beta^2\right)}{\beta}$$

M_{KDN3}-model

- A simplified version of the M_{KDN2} model.

$$M_{\mathrm{KDN2}} = \frac{1 + \frac{h_0}{LC_{f0}}\left(1-\beta^2\right)}{\beta} = \frac{1}{1-(1-\beta)} + \frac{\frac{h_0}{LC_{f0}}\left(2\left(1-\beta\right) - (1-\beta)^2\right)}{1-(1-\beta)}$$

Linearization (i): $$\frac{1}{1-(1-\beta)} \approx 1 + 1.18(1-\beta)$$

Linearization (ii): $$\frac{2\left(1-\beta\right) - (1-\beta)^2}{1-(1-\beta)} \approx 2.18(1-\beta)$$

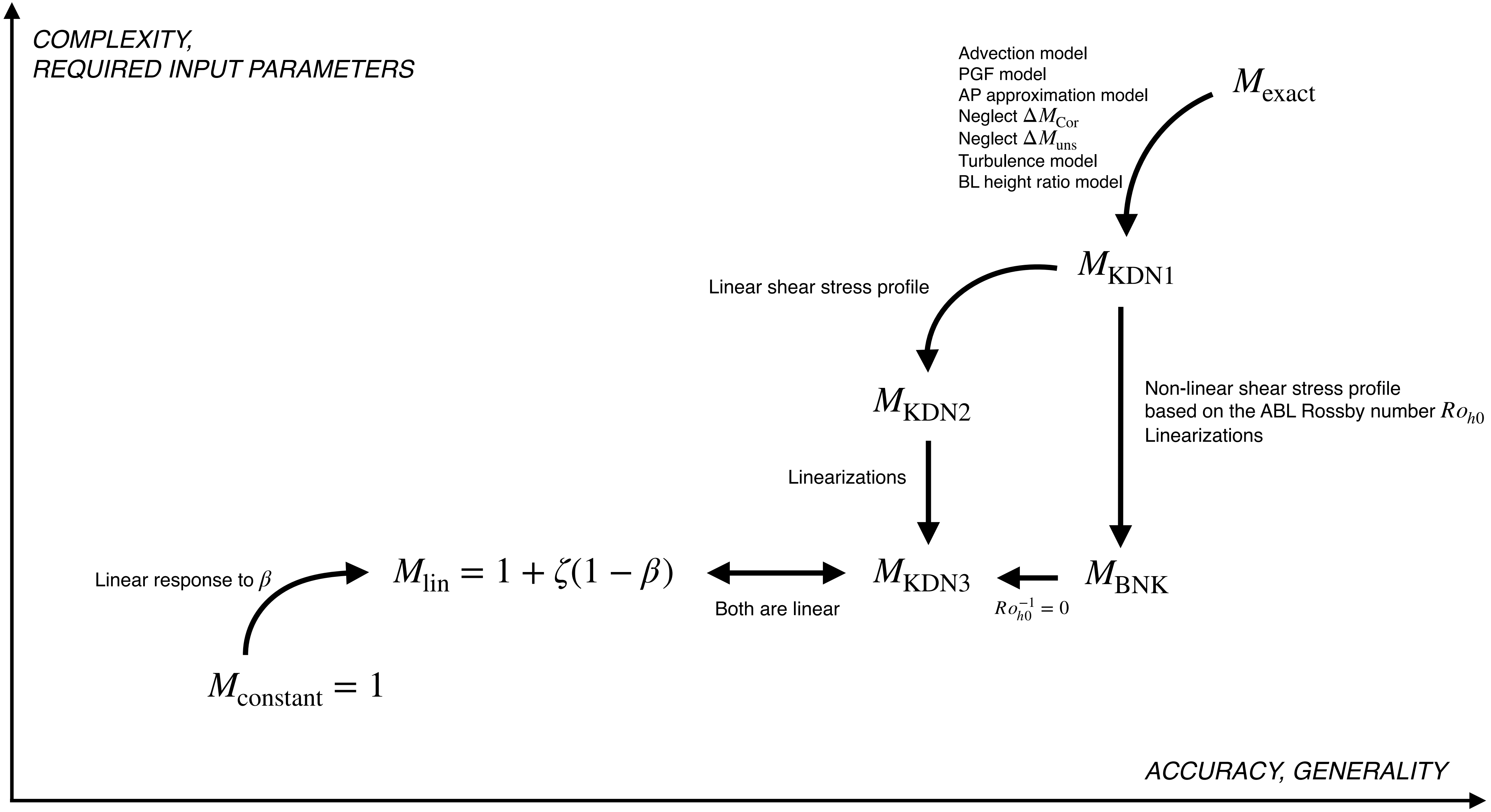
$\Rightarrow$

$$M_{\mathrm{KDN3}} = 1 + \underbrace{\left(1.18 + 2.18\frac{h_0}{LC_{f0}}\right)}_{\zeta_{\mathrm{KDN3}}}(1-\beta)$$

See Appendix of Kirby et al. (2023)

It's linear and gives an explicit expression of ζ!

Momentum availability models

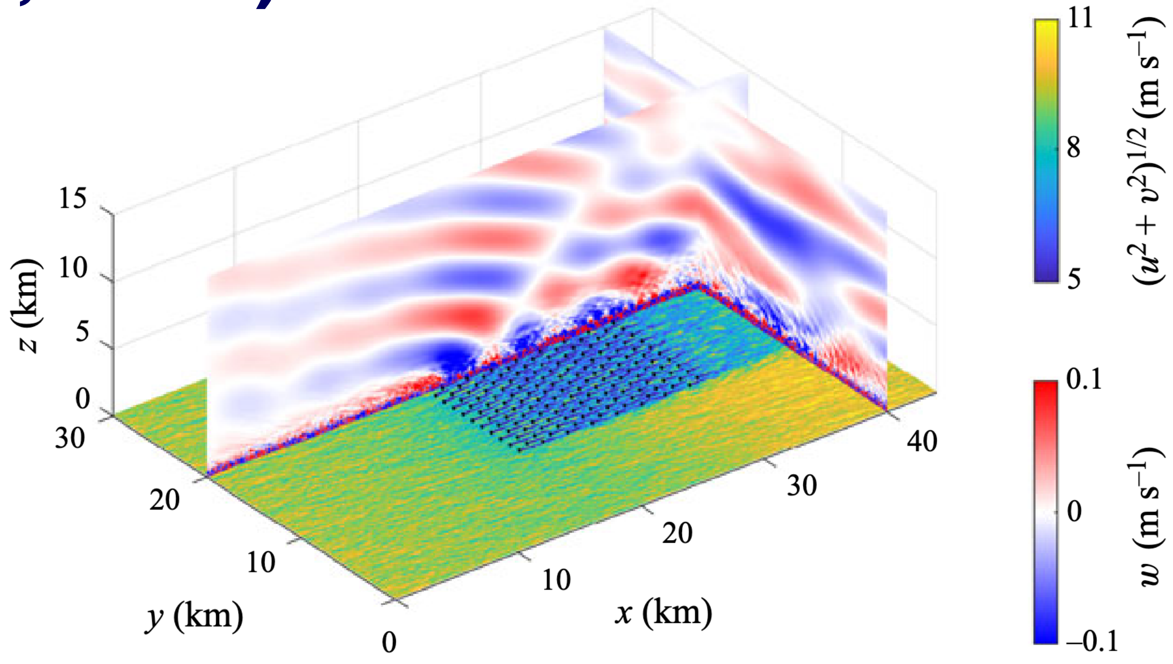


Lanzilao, L., and J. Meyers. “A Parametric Large-Eddy Simulation Study of Wind-Farm Blockage and Gravity Waves in Conventionally Neutral Boundary Layers.” *Journal of Fluid Mechanics* 979 (2024): A54. https://doi.org/10.1017/jfm.2023.1088.
Lanzilao, Luca, and Johan Meyers. “Wind-Farm Wake Recovery Mechanisms in Conventionally Neutral Boundary Layers.” *Journal of Fluid Mechanics* 1015 (2025): A5. https://doi.org/10.1017/jfm.2025.10320.

LES (Lanzilao & Meyers 2024, 2025)

LES setup:

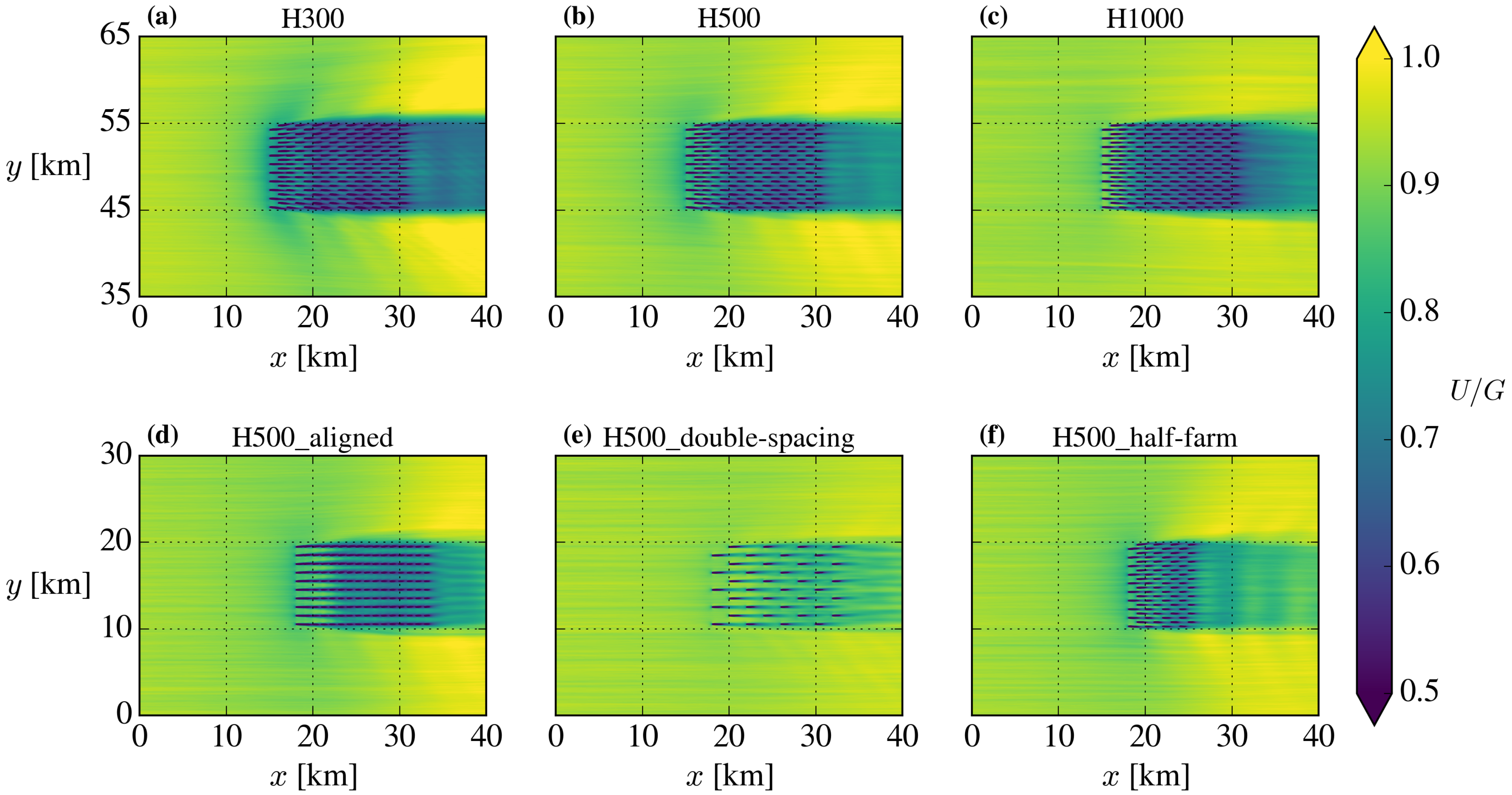
- $D = 198$ m, $z_h = 119$ m and $C_T' = 1.94$
- Conventionally neutral boundary layer (CNBL)
- $f_c = 1.14 \cdot 10^{-4}\ \mathrm{s}^{-1}$ and $z_0 = 1 \cdot 10^{-4}$ m
- $L_x \times L_y \times L_z = 110 \times 100 \times 25\ \mathrm{km}^3$
- $N_x \times N_y \times N_z = 1760 \times 2300 \times 490 \approx 2 \cdot 10^9$



Six cases:

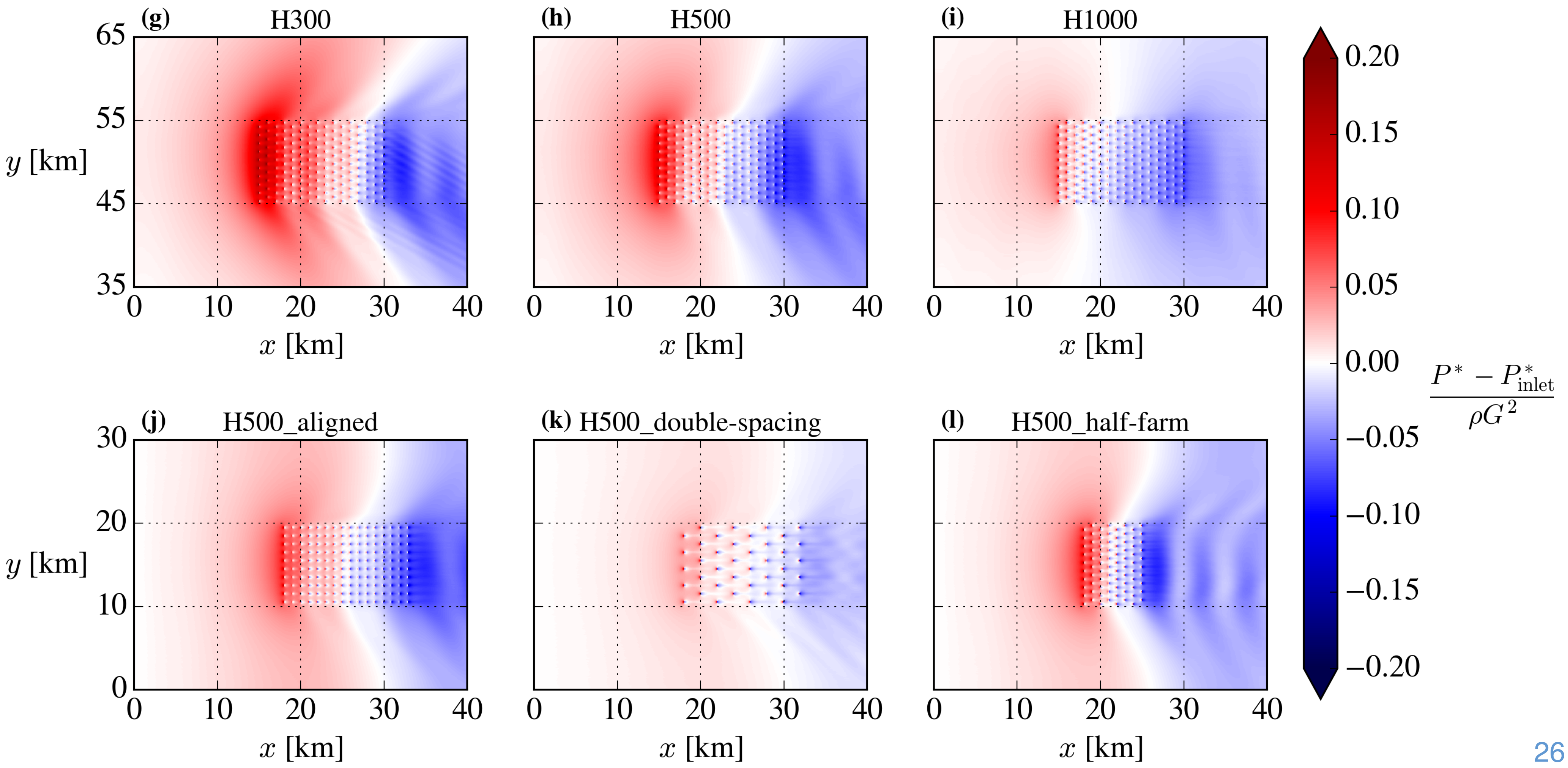
- Three cases with varying ABL height, $h_{\mathrm{ABL}} = \{300, 500, 1000\}$ m and staggered turbine layout.
- Three cases with other turbine layouts (aligned, double spacing and half farm) for $h_{\mathrm{ABL}} = 500$ m.

Streamwise velocity at hub height



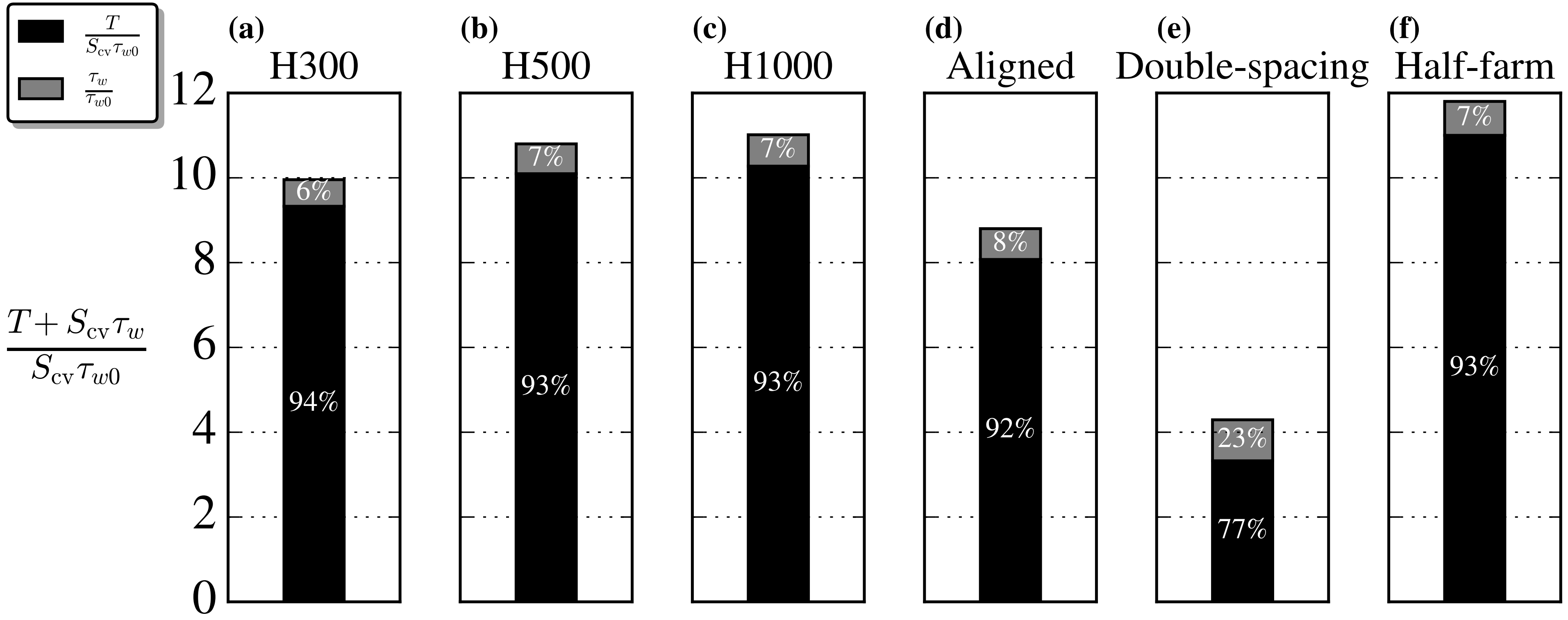
Pressure perturbation: $P^* \equiv P - P_\infty$, where P_∞ is the background pressure which drives the flow.

Pressure perturbation at hub height



Momentum sinks

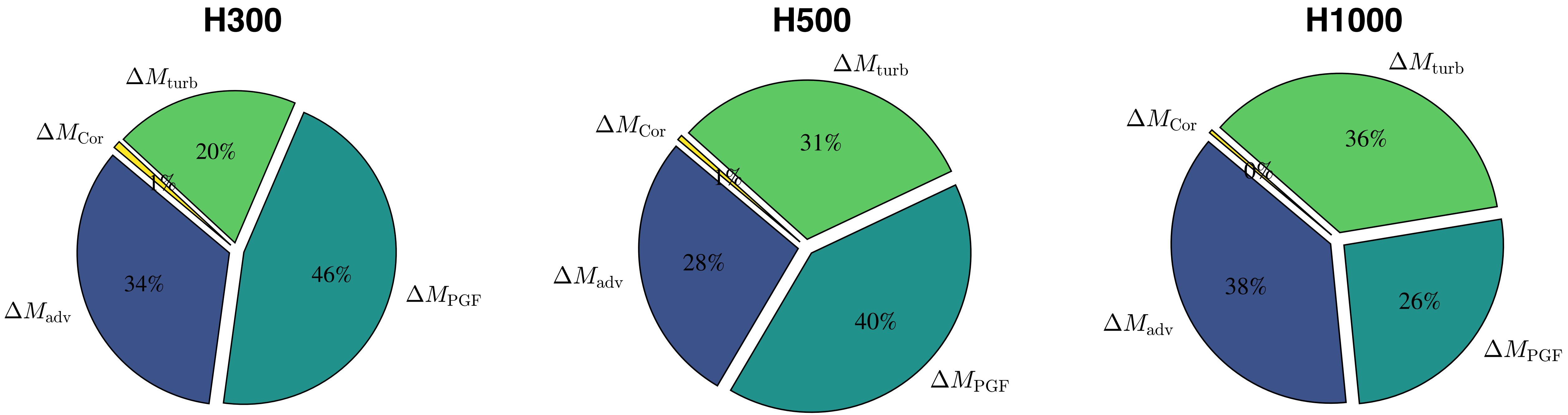
$$\underbrace{\frac{T}{S_{cv}\tau_{w0}}}_{\text{turbine drag}} + \underbrace{\frac{S_{cv}\tau_w}{S_{cv}\tau_{w0}}}_{\text{surface friction}} = M$$



Takeaway: Turbine drag dominates. The modeling of γ in $\beta^\gamma = \frac{\tau_w}{\tau_{w0}}$ is not very important.

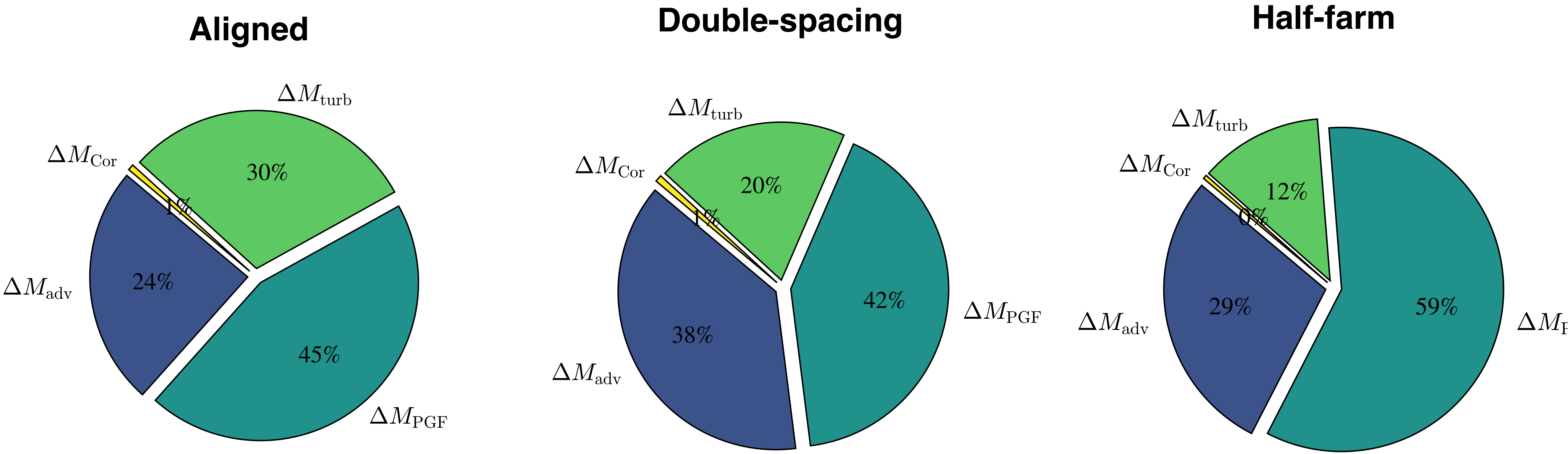
Contributions to M for varying ABL heights

$$M = 1 + \Delta M_{\text{adv}} + \Delta M_{\text{PGF}} + \Delta M_{\text{Cor}} + \Delta M_{\text{turb}} + \Delta M_{\text{uns}}$$



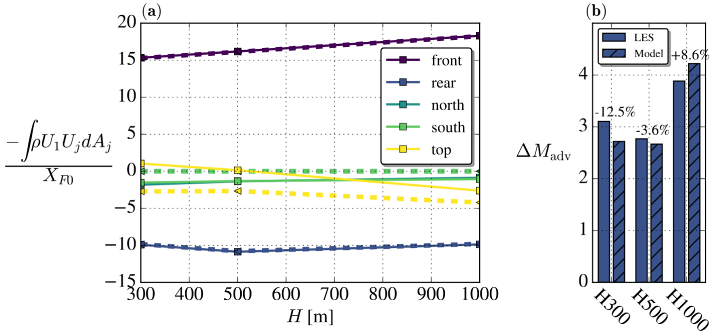
Coriolis and unsteadiness terms are negligible!

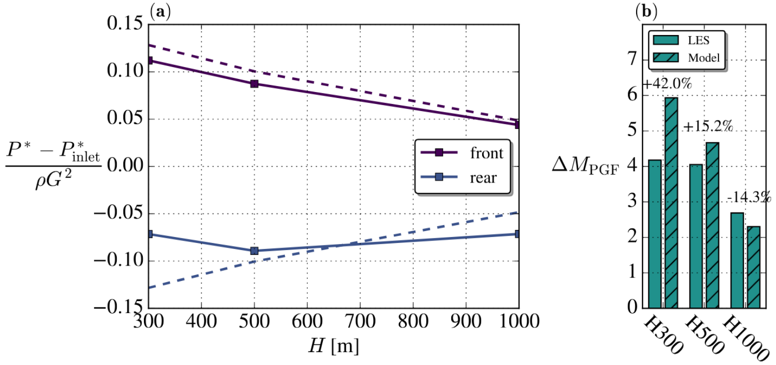
Contributions to M for varying turbine layouts

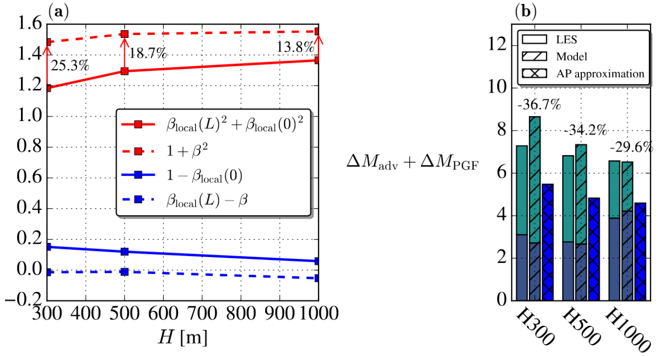


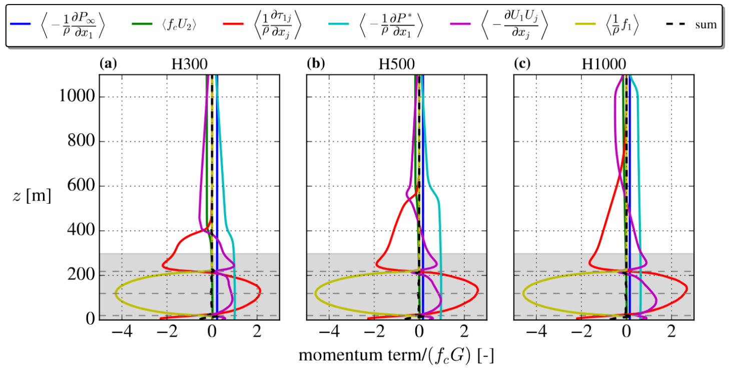
Evaluation of sub-models in the M_{KDN1}-model

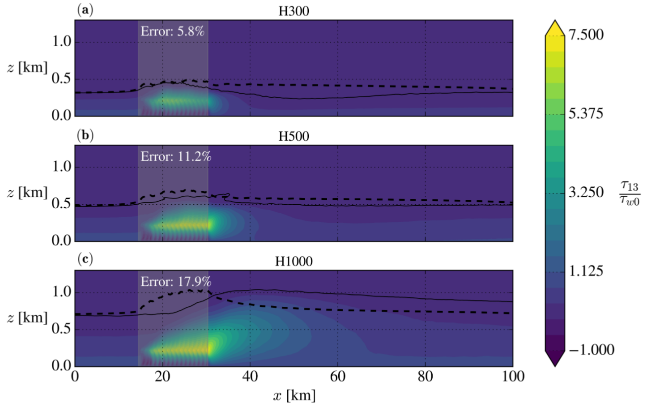
- At this point in the paper, we would go through each sub-model (i.e. ΔM_{adv}, ΔM_{PGF}, etc.) and test their underlying assumptions using LES data.
- It is a lengthy process (~10 pages), so will skip this part. See §4.2-4.8 in the paper.

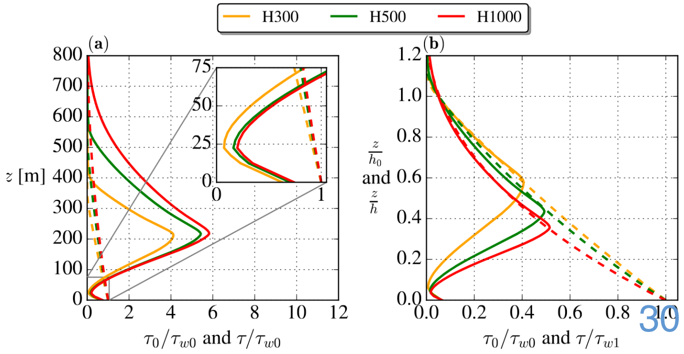




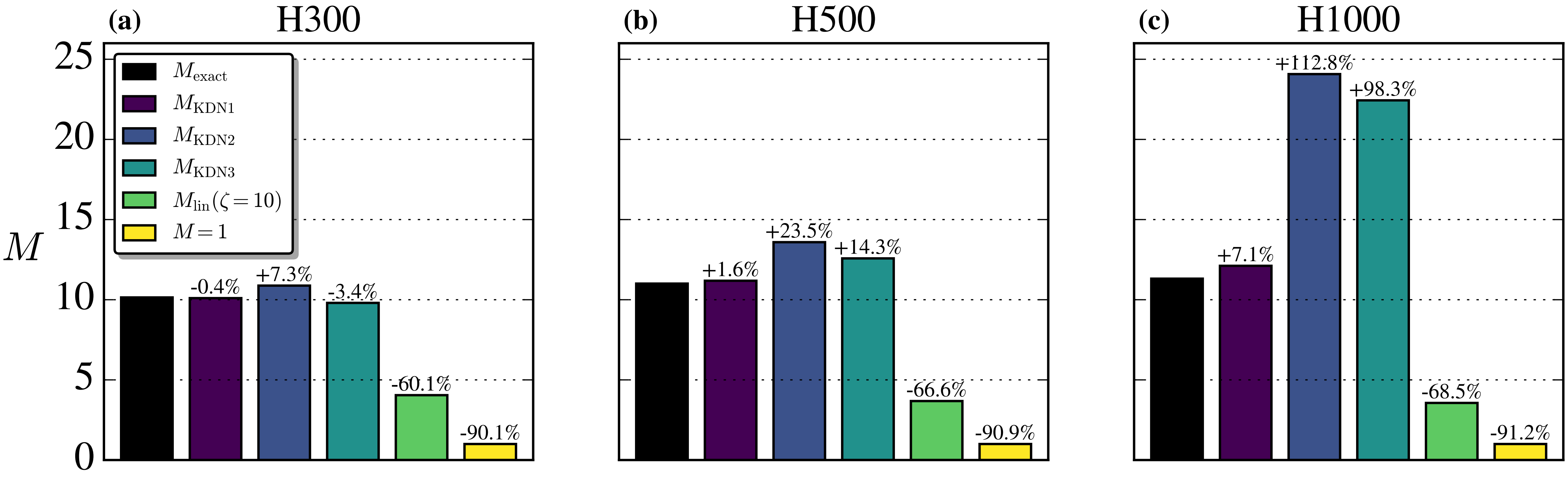






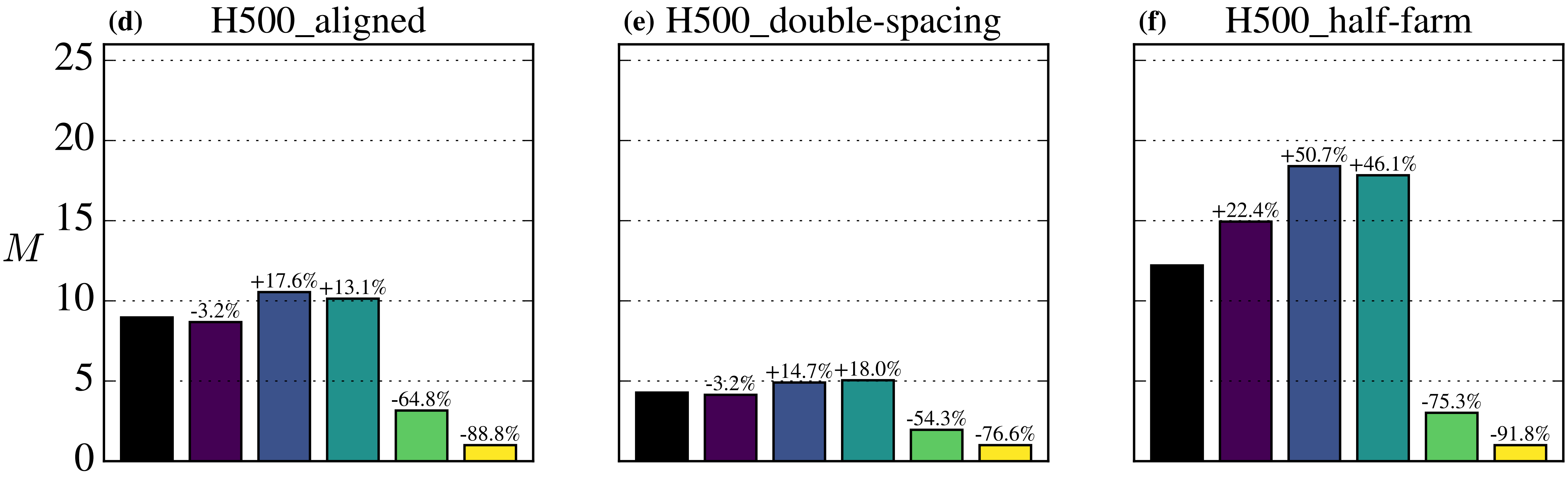


Test of M-models for varying ABL heights



Takeaway: M_{KDN1} works well for all three cases. M_{KDN2} and M_{KDN3} fail for large ABL heights.

Test of M-models for varying turbine layouts



Takeaway: Similar to H500 results —> problem with M_{KDN2} and M_{KDN3} models is mainly related to the ABL height.

3. ABL Rossby extension of an analytic momentum availability model

Improving the M_{KDN2} and M_{KDN3} models

- M_{KDN1} works well. Why bother with the M_{KDN2} and M_{KDN3} models?

  Because they are simpler and more practical.

- M_{KDN2} and M_{KDN3} work well for many cases, except tall ABLs.

Complexity ↑

$$M_{\mathrm{KDN1}} = \frac{1 + \frac{H_F}{LC_{f0}}\left(1-\beta^2\right) - \frac{\tau_{t0}}{\tau_{w0}}}{\beta\left(1 - \frac{\tau_{t0}}{\tau_{w0}}\right)}$$

$$M_{\mathrm{KDN2}} = \frac{1 + \frac{h_0}{LC_{f0}}\left(1-\beta^2\right)}{\beta}$$

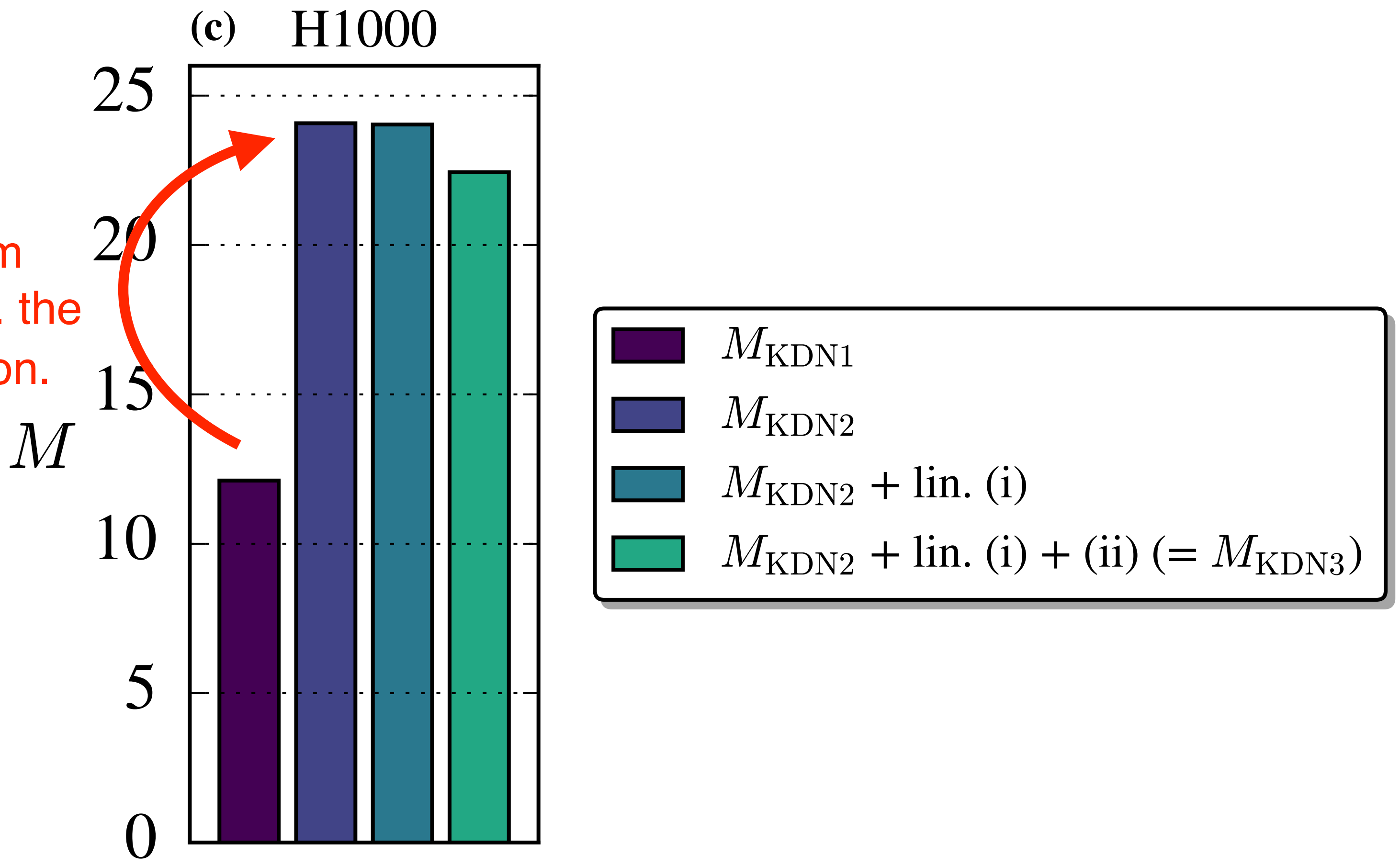
$$M_{\mathrm{KDN3}} = 1 + \underbrace{\left(1.18 + 2.18\frac{h_0}{LC_{f0}}\right)}_{\zeta_{\mathrm{KDN3}}}(1-\beta)$$

M_{KDN2} and M_{KDN3} problem for tall ABLs

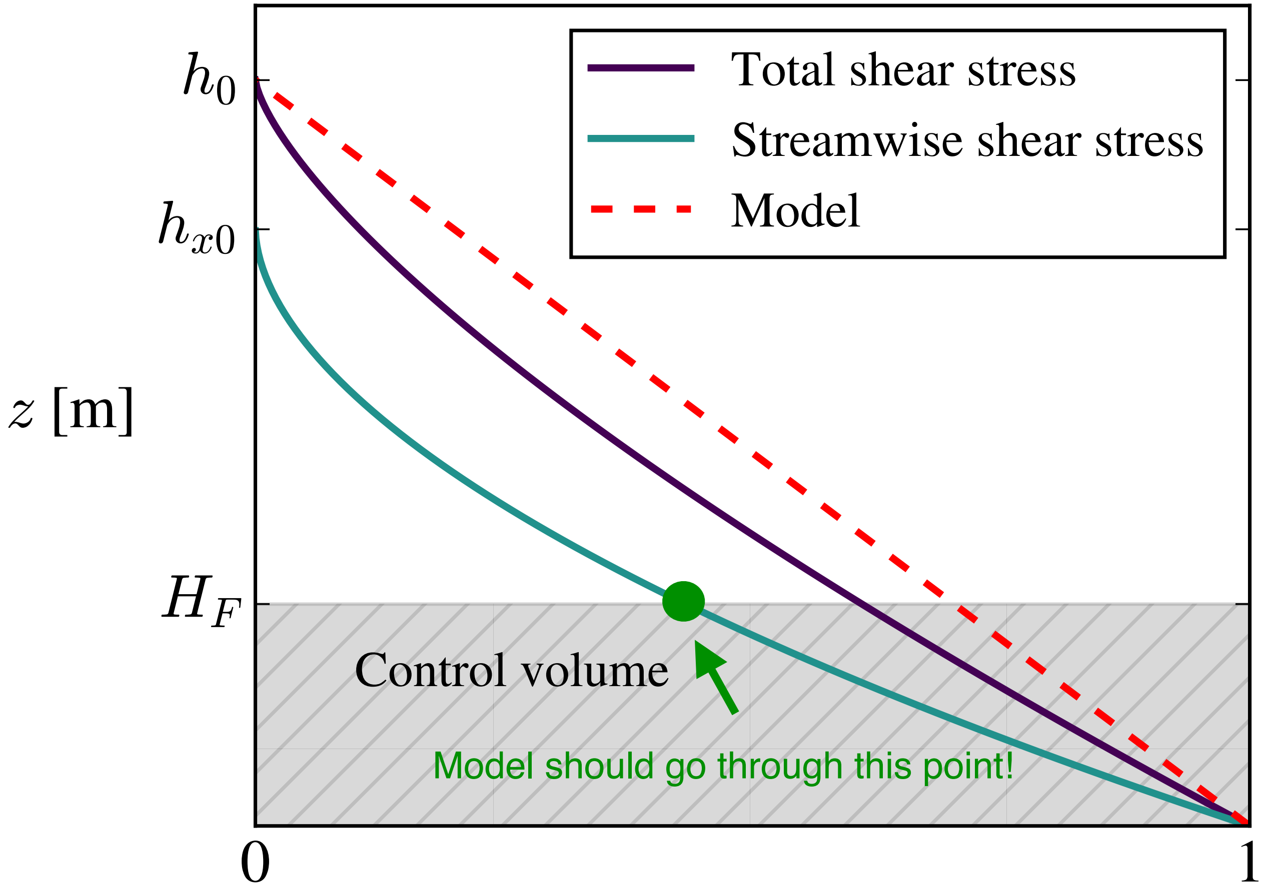
Main error clearly comes from the M_{KDN2} simplification, i.e. the linear shear stress assumption.

Linear shear stress assumption:

$$1 - \frac{\tau_{xt0}}{\tau_{xw0}} \approx \frac{H_F}{h_0}$$



Problems with linear shear stress assumption



Linear shear stress assumption:

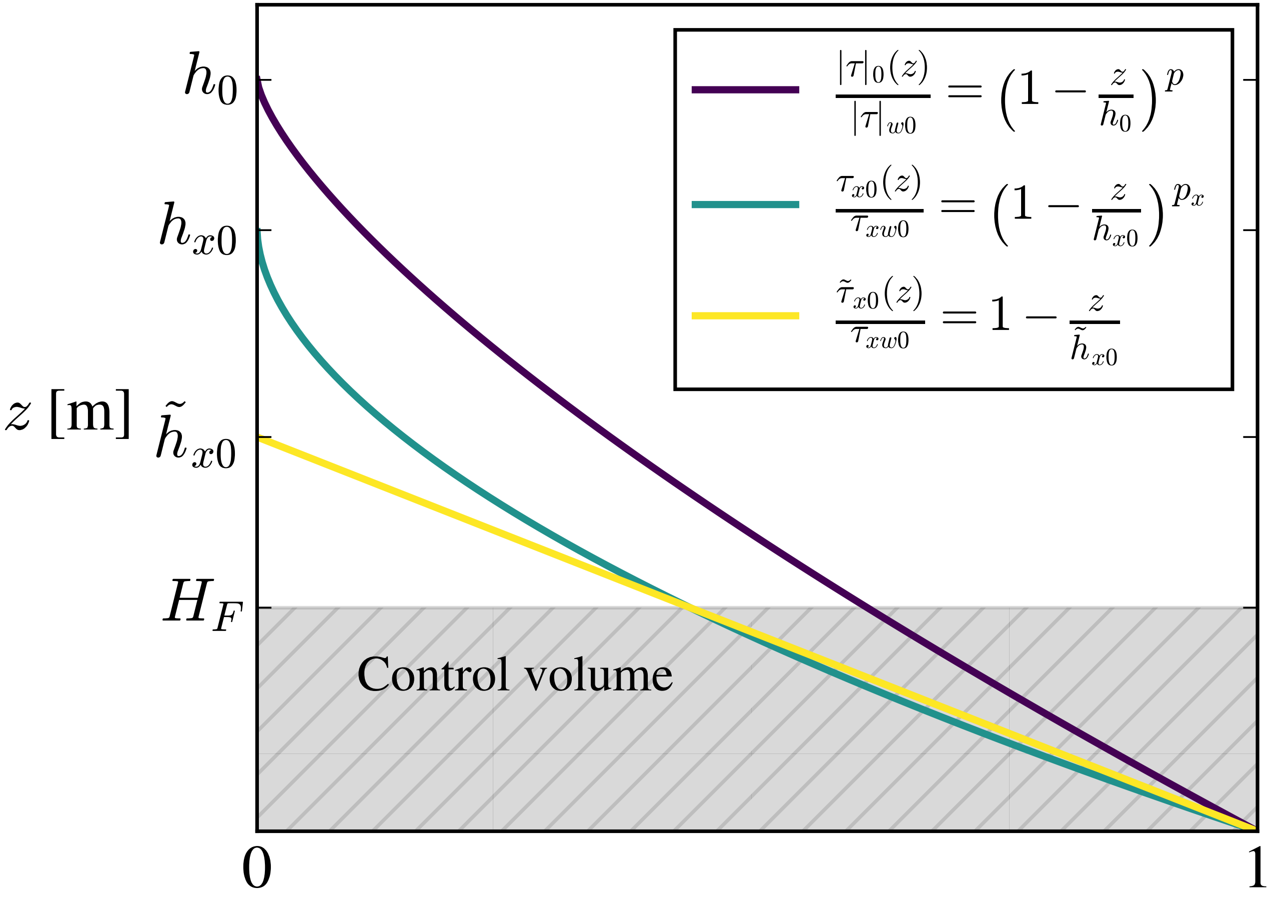
$$\frac{\tau_{x0}(z)}{\tau_{xw0}} = 1 - \frac{z}{h_0}$$

$$\Rightarrow 1 - \frac{\tau_{xt0}}{\tau_{xw0}} \approx \frac{H_F}{h_0}$$

Two problems:

1. The ABL height, h_0, is connected to the total shear stress, not streamwise shear stress.
2. The shear stress profile, $\tau_{x0}(z)$, is not linear, but slightly concave.

A fictive ABL height $\tilde{h}_{x0}$



Solution:
Define a fictive ABL height $\tilde{h}_{x0}$ and profile $\tilde{\tau}_{x0}(z)$, which satisfy $\tilde{\tau}_{x0}(H_F) = \tau_{xt0}$.

New model:

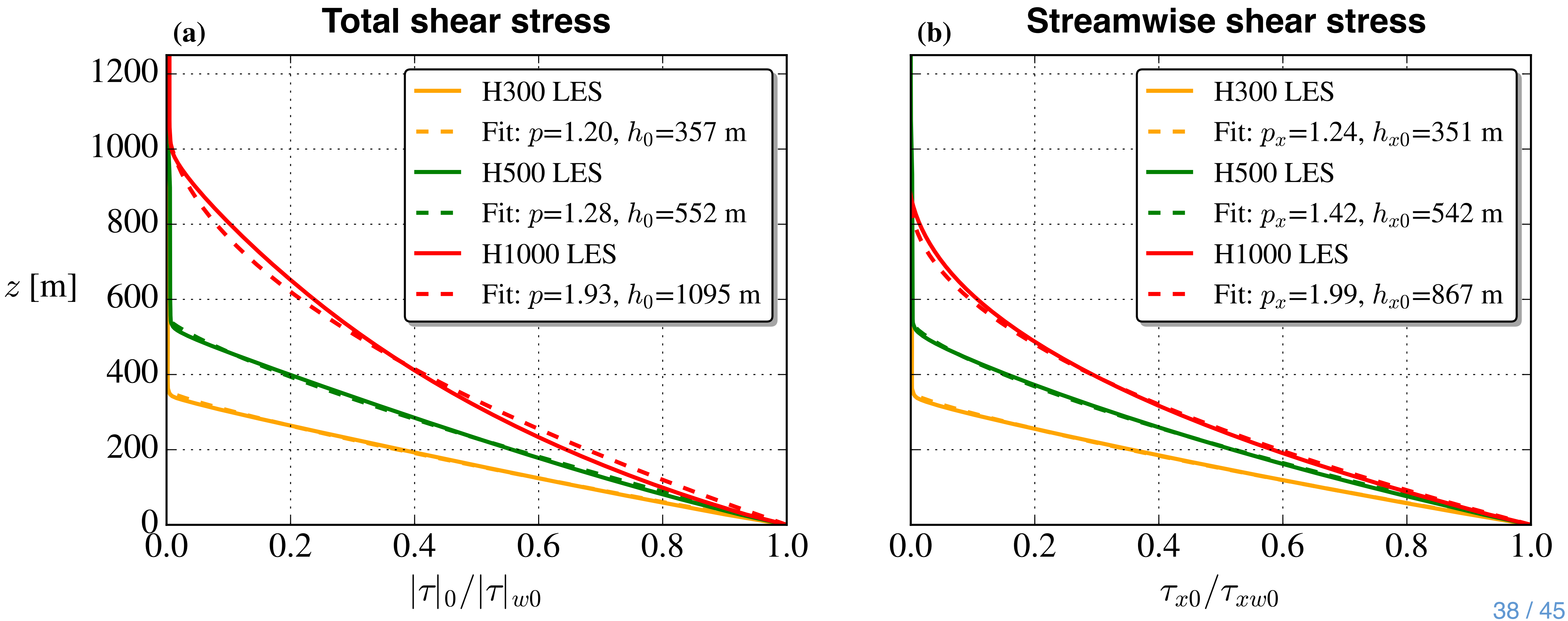
$$1 - \frac{\tau_{xt0}}{\tau_{xw0}} \approx \frac{H_F}{\tilde{h}_{x0}}$$

The problem is now converted to a problem of finding $\tilde{h}_{x0}$.

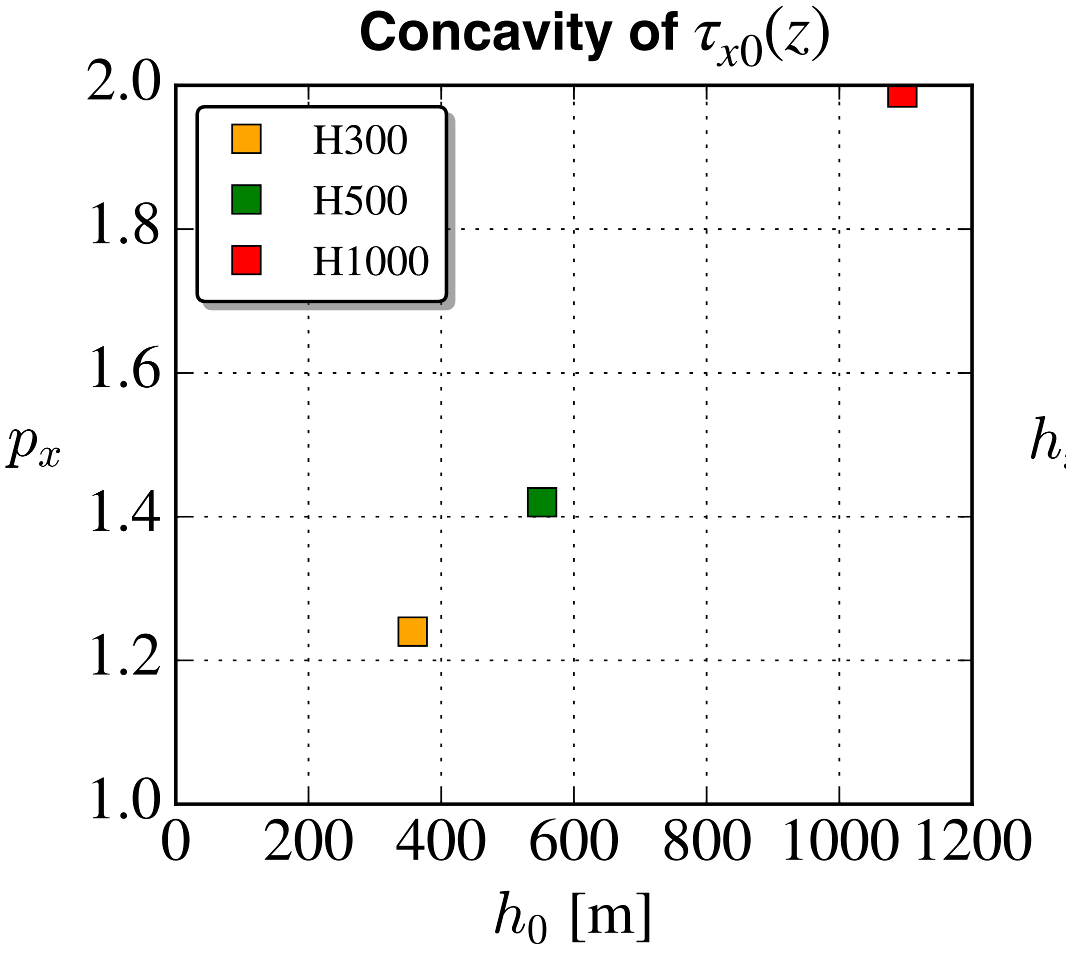
If the streamwise shear stress profile $\tau_{x0}(z)$ is a power function, one obtains:

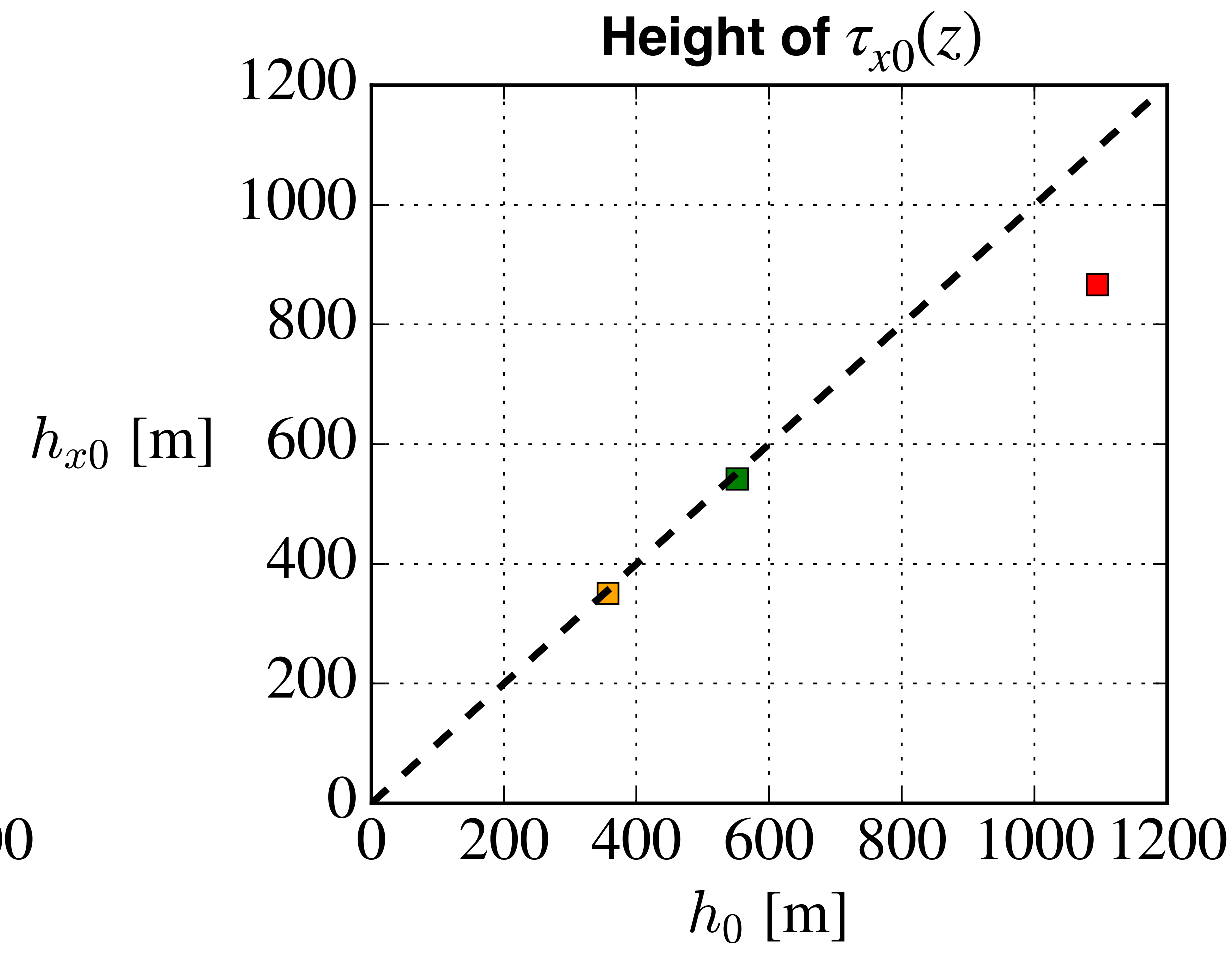
$$\tilde{h}_{x0} \approx H_F + p_x^{-1.25}\left(h_{x0} - H_F\right)$$

Fit of shear stress profiles of CNBLs



Trends of h_{x0} and p_x





General Rossby number definition: $Ro = \frac{\mathscr{U}}{f_c \mathscr{L}}$

ABL Rossby number, $Ro_{h0} = G/(f_c h_0)$

Motivation for Ro_{h0}:

Small $h_0 \Rightarrow p_x = 1$ *and* $h_{x0}/h_0 \to 1$.

Small $f_c \Rightarrow p_x = 1$ *and* $h_{x0}/h_0 \to 1$.

They have the same effect! Use Ro_{h_0}.

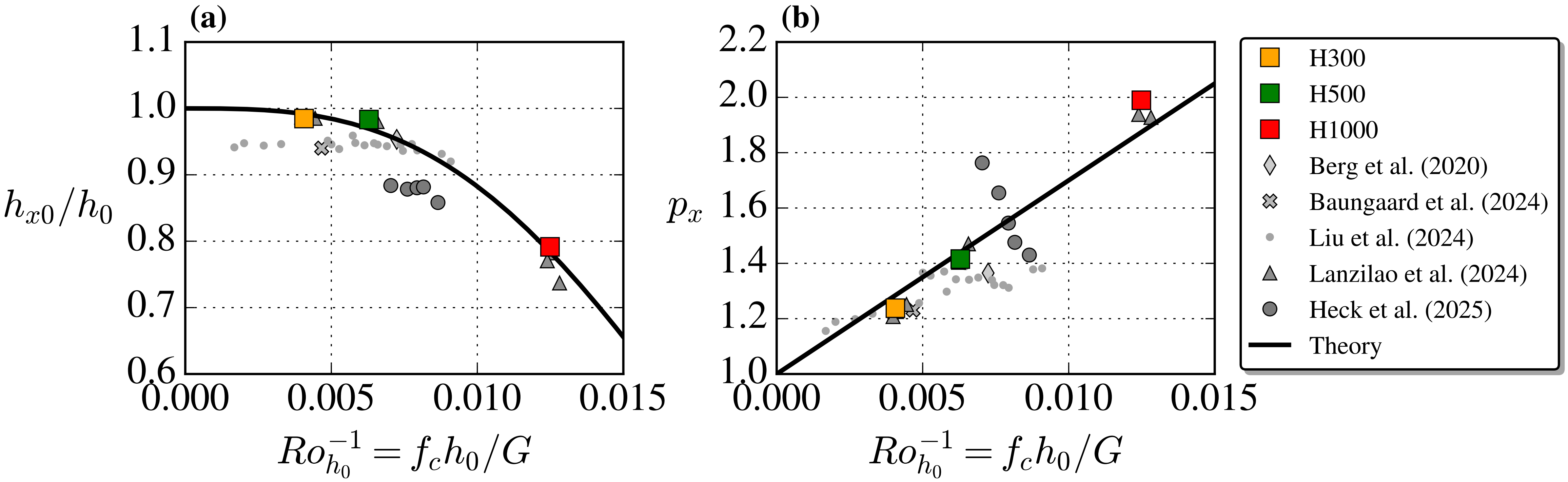
- van der Laan et al. (2020) found that their Apsley-type ABL profiles collapse upon two Rossby numbers, $Ro_0 = G/(f_c z_0)$ and $Ro_\ell = G/(f_c \ell_{\max})$.
- Liu et al. (2024): “The dynamics in CNBLs are governed by Ro and Zi”.

Parametrization of h_{x0}/h_0 and p_x

Theoretical model:

$$\frac{h_{x0}}{h_0} = \exp\left(-\left(\frac{Ro_{h0}^{-1}}{0.02}\right)^3\right)$$

$$p_x = 1 + 70 Ro_{h0}^{-1}$$



$$\tilde{h}_{x0} = H_F + \left(1 + 70 Ro_{h0}^{-1}\right)^{-1.25} \left(h_0 \exp\left(-\left(\frac{Ro_{h0}^{-1}}{0.02} \right)^3 \right) - H_F \right)$$

The M_{BNK}-model

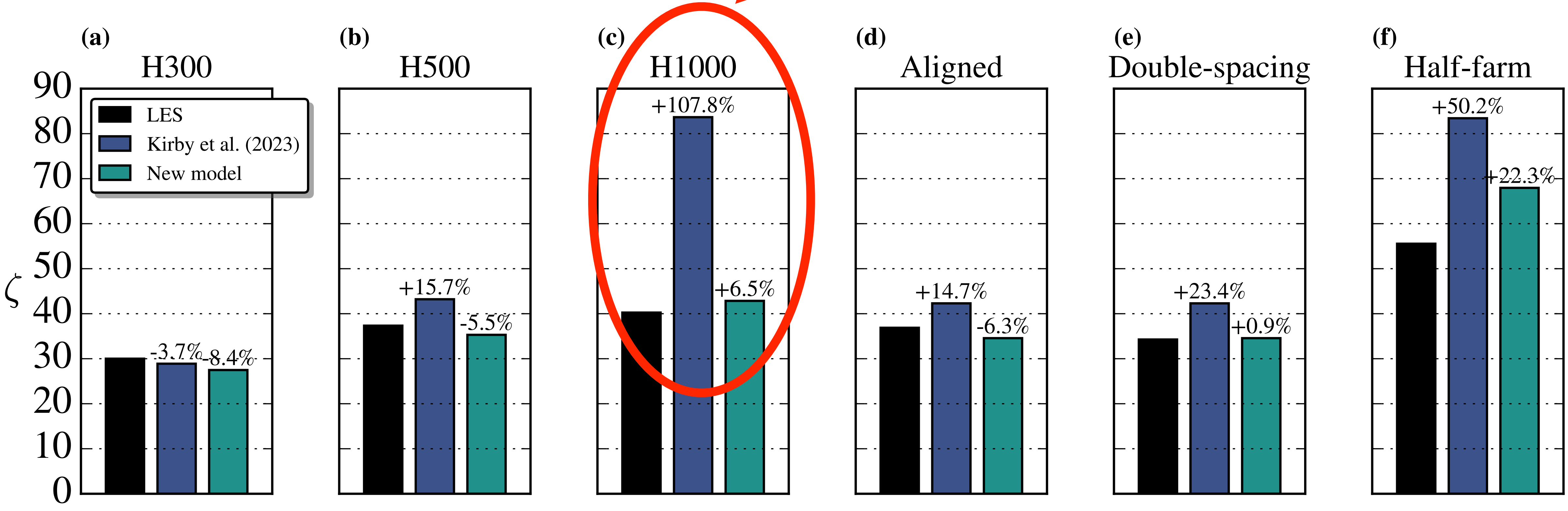
- We call our new model M_{BNK}.

$$M_{\text{BNK}} = 1 + \underbrace{\left(1.18 + 2.18 \frac{\tilde{h}_{x0}}{LC_{f0}} \right)}_{\zeta_{\text{BNK}}} (1 - \beta)$$

- It is linear in β and only have one additional parameter Ro_{h0} compared to the M_{KDN3} model.
- For the special case $Ro_{h0}^{-1} = 0$, the model is equivalent to M_{KDN3}, hence M_{BKN} can also be viewed as an extension of M_{KDN3}.

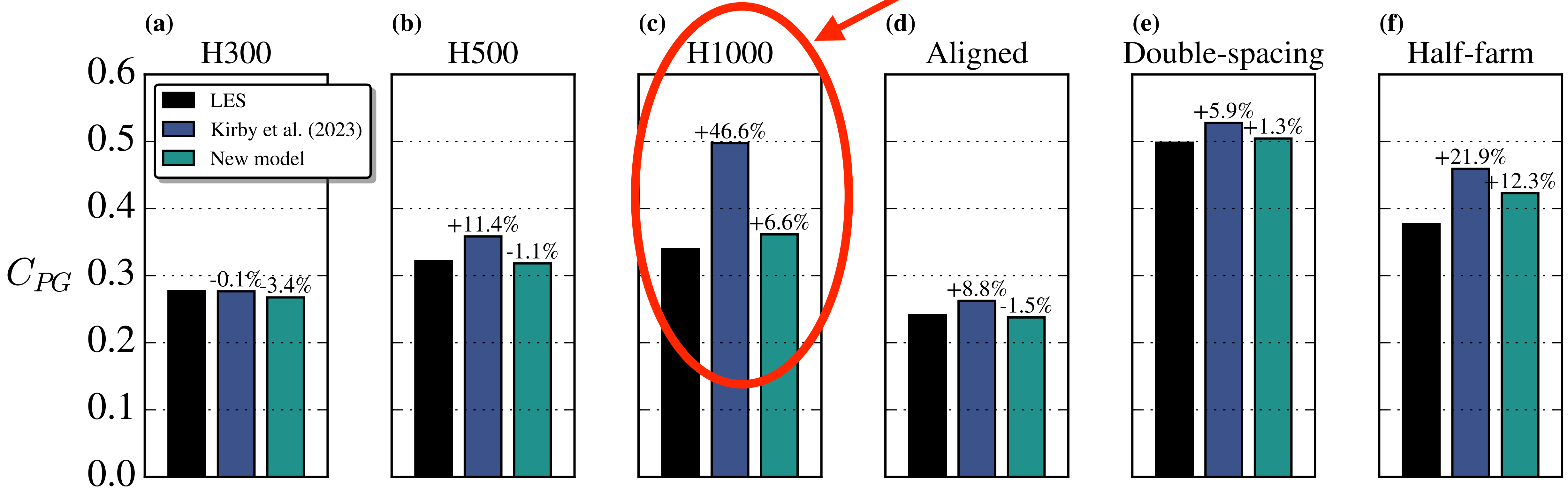
Comparison of ζ

Major improvement for the tall ABL case.
Still good performance for the other cases.



Comparison of C_{PG} predictions

Again, major improvement for the tall ABL case.



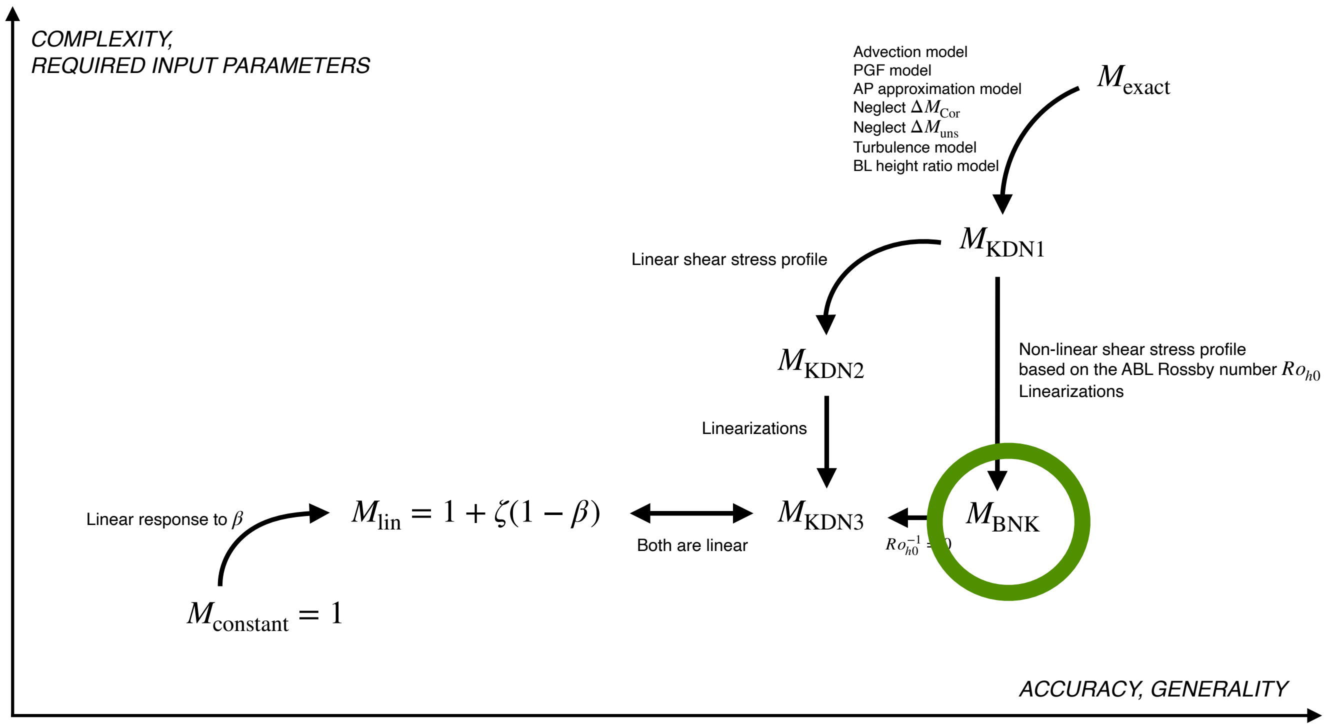
Prediction of C_{PG} is now within -4 to 7 % for five of the six cases.

Conclusions

1. One needs a momentum availability model, M, in the two-scale momentum theory. It is important for accurate wind farm efficiency prediction.

2. We have introduced the M_{BNK}-model, which is a simple and practical model. It includes the effect of shear stress veering through Ro_{h0}.

3. M_{BNK} improves the model predictions significantly for tall ABLs compared to M_{KDN3} and also maintains the good prediction for the other considered cases.



arXiv link: https://arxiv.org/abs/2602.10126

Extra slides

Research questions

- How accurate are the various M-models?
- How accurate are the sub-models/assumptions in the M_{KDN}-models?
- Is the farm thrust or surface friction larger?
- Which is the most important physical mechanism for M? I.e. which ΔM_{xxx}-term?

Can answer with LES data of $U_i(x, y, z)$, $P(x, y, z)$ *and* $\tau_{ij}(x, y, z)$ *for* $x_i \in V_{\mathrm{cv}}$.

History of two-scale momentum theory

2016	Initial idea	Nishino, T. "Two-Scale Momentum Theory for Very Large Wind Farms." *Journal of Physics: Conference Series* 753 (2016): 032054. https://doi.org/10.1088/1742-6596/753/3/032054.
2019	Effect of support structure	Ma, Lun, Takafumi Nishino, and Antonios F. Antoniadis. "Prediction of the Impact of Support Structures on the Aerodynamic Performance of Large Wind Farms." *Journal of Renewable and Sustainable Energy* 11, no. 6 (2019): 063306. https://doi.org/10.1063/1.5120602.
2020	Formal formulation	Nishino, T., and T. D. Dunstan. "Two-Scale Momentum Theory for Time-Dependent Modelling of Large Wind Farms." *Journal of Fluid Mechanics* 894 (2020): A2. https://doi.org/10.1017/jfm.2020.252.
2021	NWP test of linear M	Patel, K., T. D. Dunstan, and T. Nishino. "Time-Dependent Upper Limits to the Performance of Large Wind Farms Due to Mesoscale Atmospheric Response." *Energies* 14, no. 19 (2021): 6437. https://doi.org/10.3390/en14196437.
2022	Suite of 50 infinite wind farm LESs*	Kirby, A., T. Nishino, and T. D. Dunstan. "Two-Scale Interaction of Wake and Blockage Effects in Large Wind Farms." *Journal of Fluid Mechanics* 953 (2022): A39. https://doi.org/10.1017/jfm.2022.979.

*has two main contributions: 1) infinite to finite farm correction, and 2) concept of farm- and turbine-scale losses.

UNIVERSITY OF OXFORD

2023

Focus on fluids on 2022 paper

Stevens, R. J. A. M. "Understanding Wind Farm Power Densities." *Journal of Fluid Mechanics* 958 (2023): F1. https://doi.org/10.1017/jfm.2023.113.

Data-driven C_T^* model

Kirby, Andrew, François-Xavier Briol, Thomas D. Dunstan, and Takafumi Nishino. "Data-driven Modelling of Turbine Wake Interactions and Flow Resistance in Large Wind Farms." *Wind Energy* 26, no. 9 (2023): 968–84. https://doi.org/10.1002/we.2851.

This presentation mostly related to

Analytical M model

Kirby, A., T. D. Dunstan, and T. Nishino. "An Analytical Model of Momentum Availability for Predicting Large Wind Farm Power." *Journal of Fluid Mechanics* 976 (2023): A24. https://doi.org/10.1017/jfm.2023.844.

2024

Engineering applications

Legris, Leila, Morten Lindholt Pahus, Takafumi Nishino, and Edgar Perez-Campos. "Prediction and Mitigation of Wind Farm Blockage Losses Considering Mesoscale Atmospheric Response." *Energies* 16, no. 1 (2022): 386. https://doi.org/10.3390/en16010386.

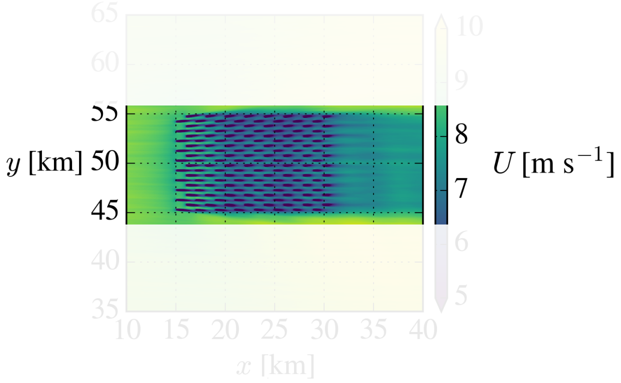
Pahus, M. L., T. Nishino, A. Kirby, and C. R. Vogel. "Control Co-Design of a Large Offshore Wind Farm Considering the Effect of Wind Extractability." *Journal of Physics: Conference Series* 2767, no. 9 (2024): 092026. https://doi.org/10.1088/1742-6596/2767/9/092026.

2025

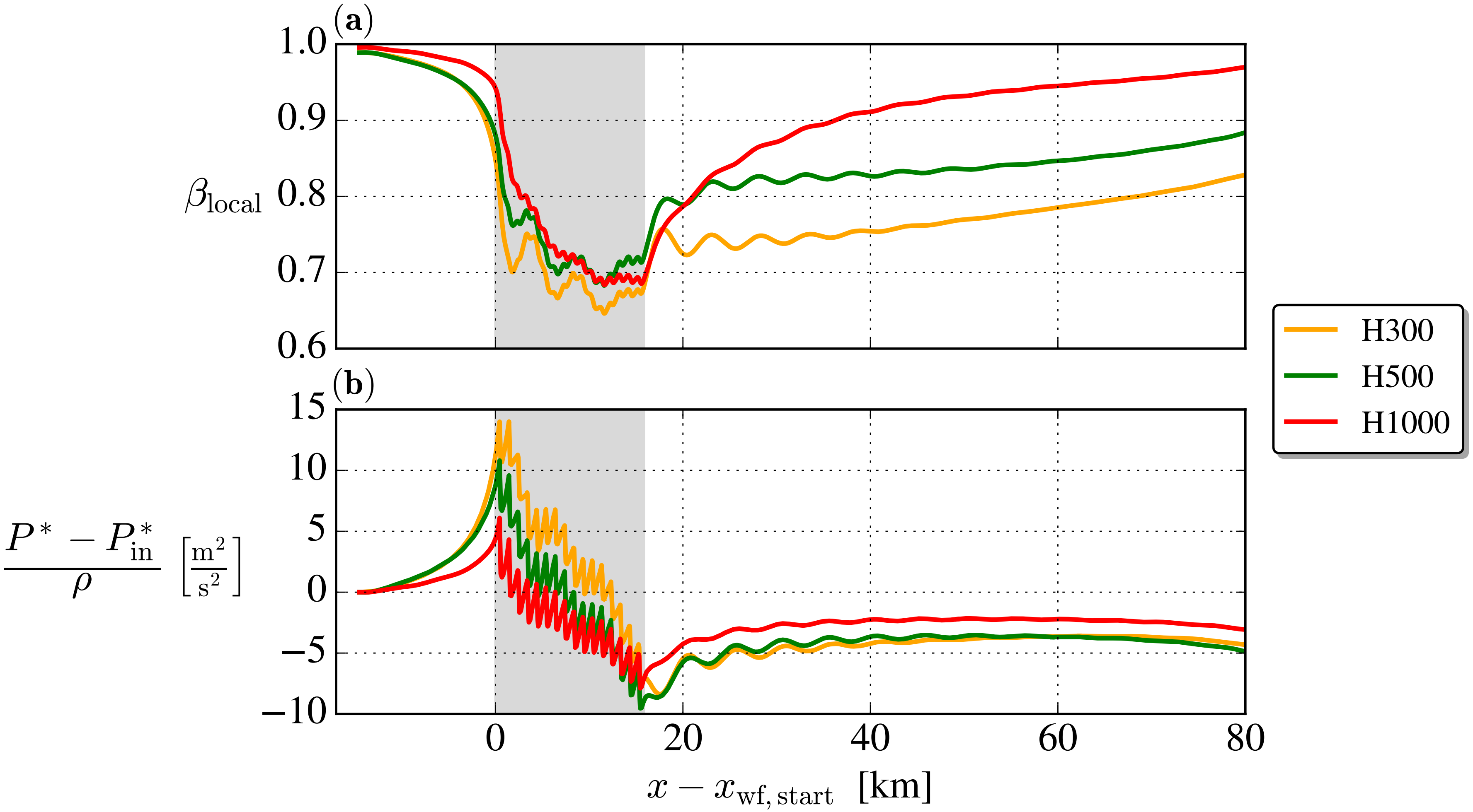
Turbine- and farm-scale losses

Kirby, A., T. Nishino, L. Lanzilao, T. D. Dunstan, and J. Meyers. "Turbine- and Farm-Scale Power Losses in Wind Farms: An Alternative to Wake and Farm Blockage Losses." *Wind Energy Science* 10, no. 2 (2025): 435–50. https://doi.org/10.5194/wes-10-435-2025.

Local wind-speed reduction: $\beta_{\text{local}} \equiv \frac{1}{H_F W U_{F0}} \int_0^{H_F} \int_{-W/2}^{W/2} U dy dz$

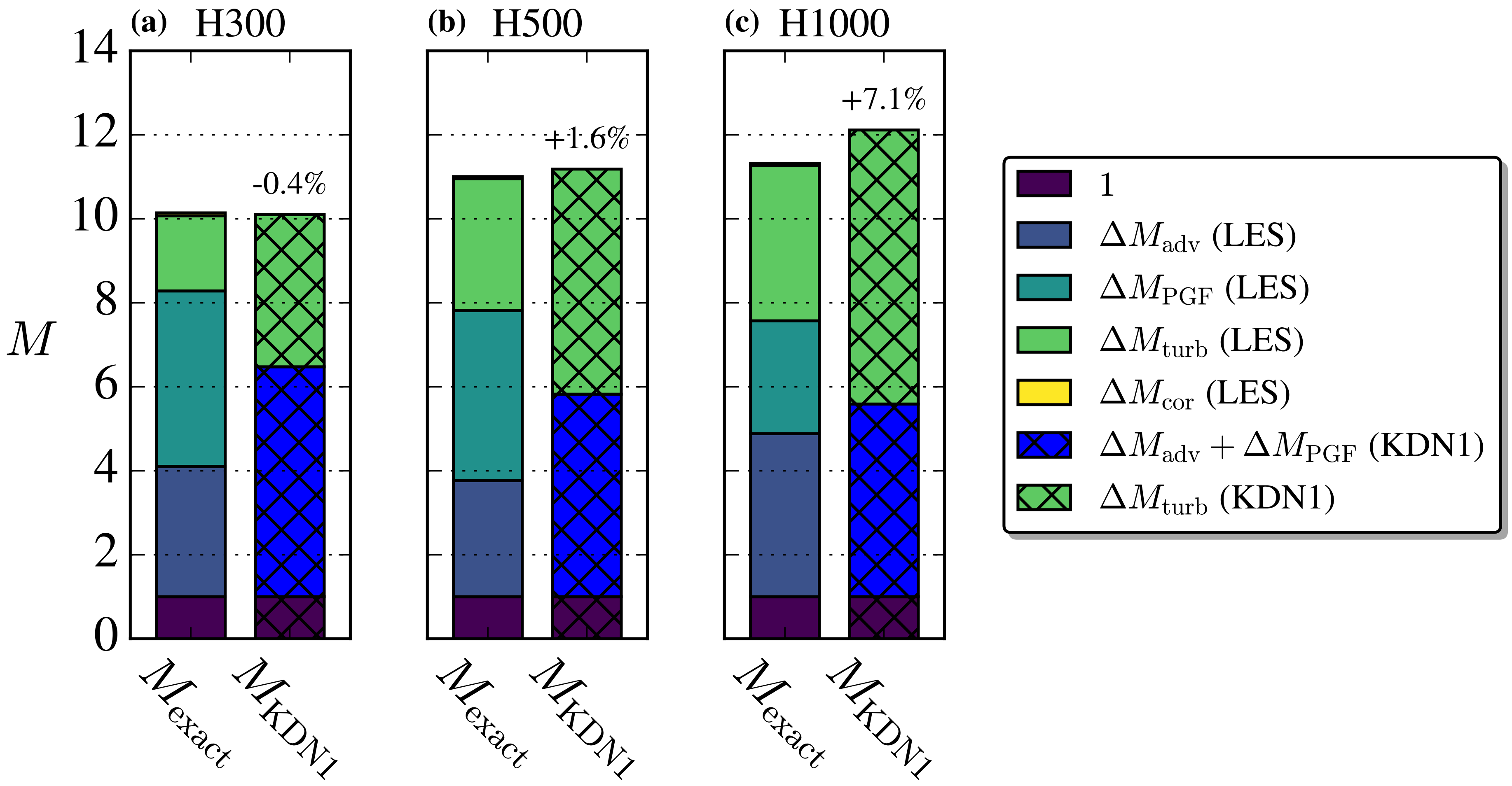


LES spanwise- and vertically-averaged



Conclusion: Overall good model prediction (within -1 to 7 %), but partly due error cancellation.

Summary of M_{KDN1}-submodel evaluations



ΔM_{adv}-model

Model:

$\Delta M_{\text{adv,north}} = \Delta M_{\text{adv,south}} = \Delta M_{\text{adv,bottom}} = 0$

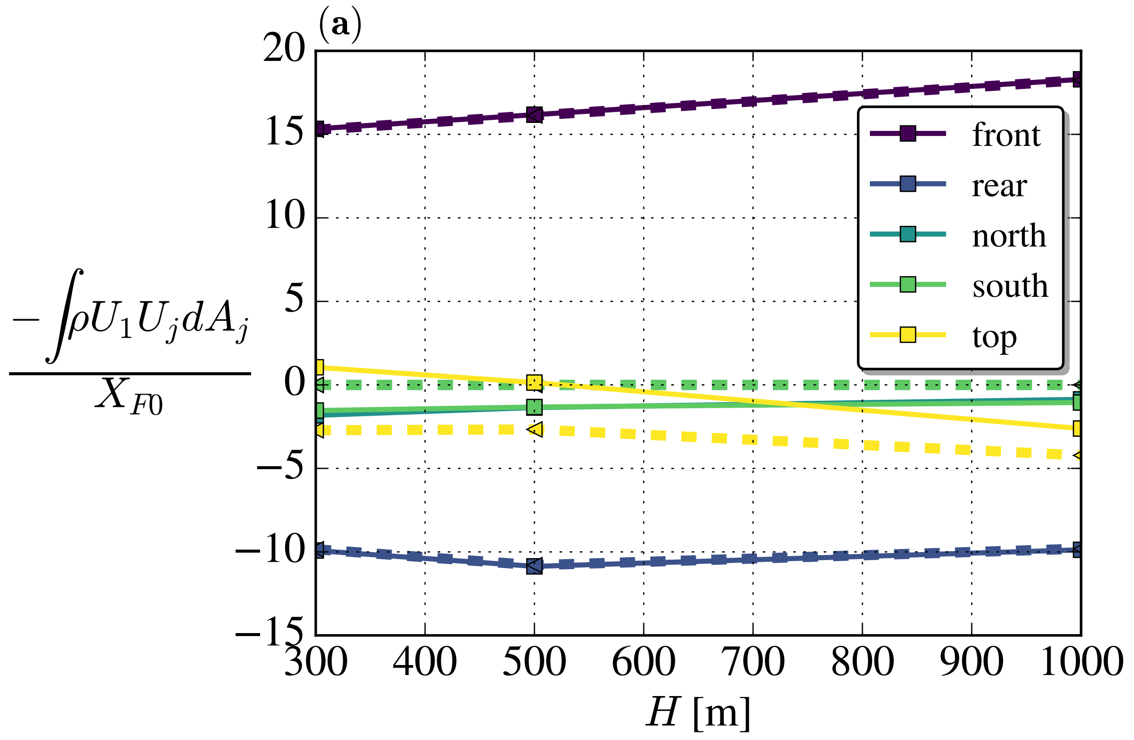
$\Delta M_{\text{adv,front}}$ and $\Delta M_{\text{adv,rear}}$: exact.

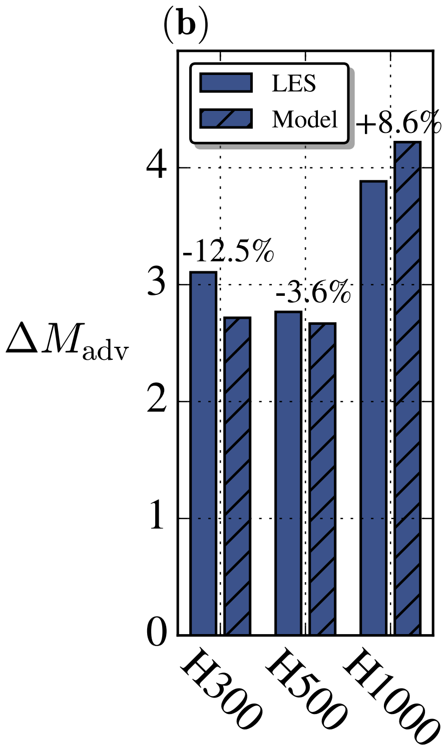
$$\Delta M_{\text{adv,top}} = \frac{H_F\left(\beta_{\text{local}}(L)^2 - \beta_{\text{local}}(0)^2\right)}{LC_{f0}}$$

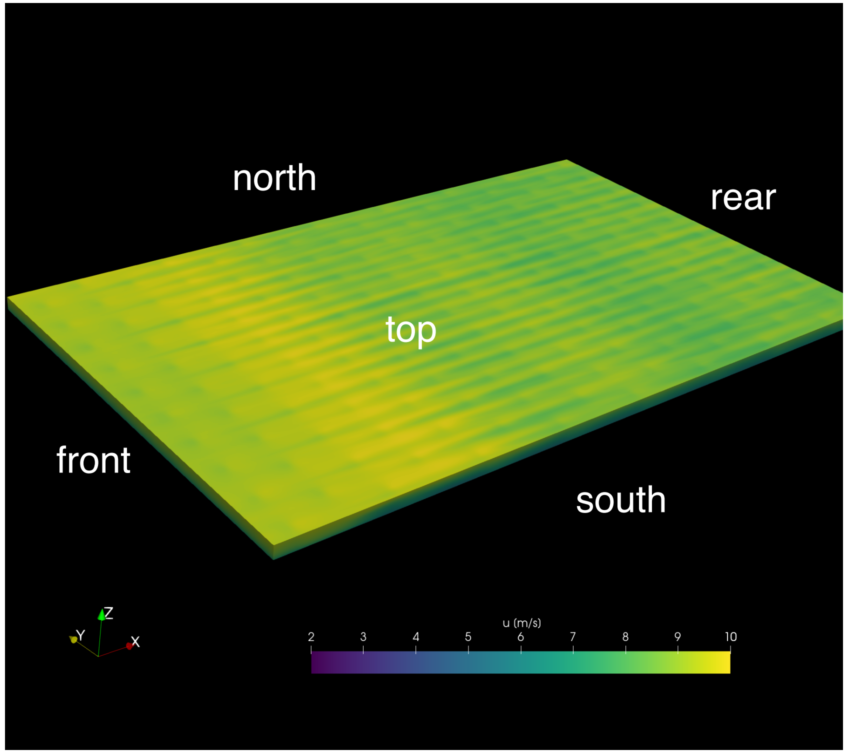
$$\Delta M_{\text{adv}} \equiv \frac{X_{\text{adv}} - X_{\text{adv0}}}{X_{F0}}$$

$$X_{\text{adv}} \equiv -\int_{\Omega_{cv}} \rho U_1 U_j dA_j$$

$$X_{\text{adv0}} = 0$$







Positive: advection *into* CV
Negative: advection *out of* CV

UNIVERSITY OF OXFORD

ΔM_{PGF}-model

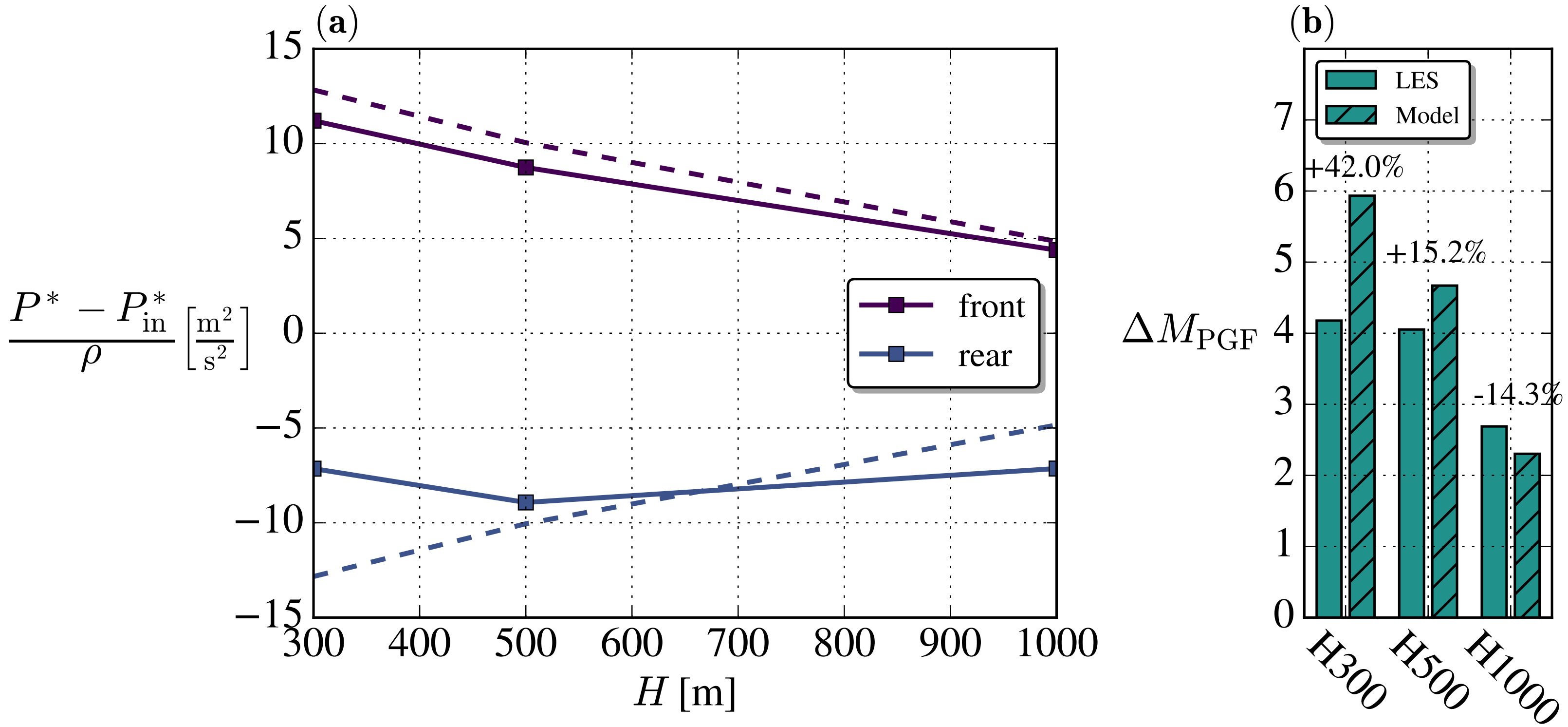
Model:

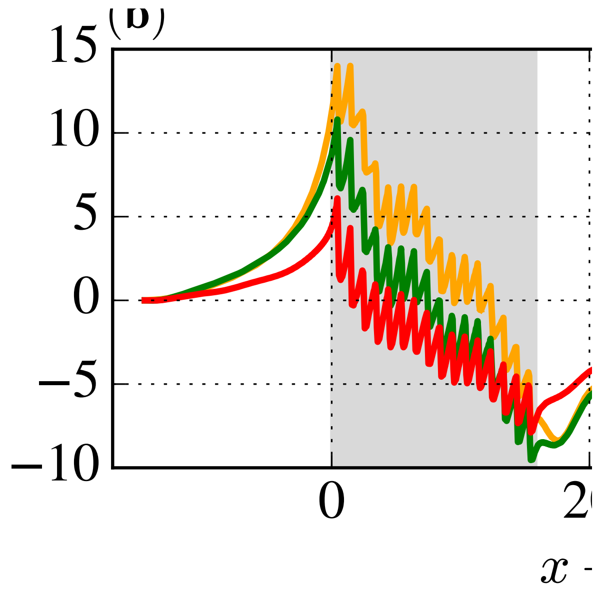
$$P^*_{\text{front}} - P^*_{\text{in}} = \frac{1}{2}\rho U_{F0}^2 \left(1 - \beta_{\text{local}}(0)^2\right)$$

$$P^*_{\text{rear}} - P^*_{\text{in}} = -\left(P^*_{\text{front}} - P^*_{\text{in}}\right)$$

$$P^* \equiv P - P_\infty$$

$$\Delta M_{\text{PGF}} = -\frac{H_F}{L\tau_{w0}}\left(P^*_{\text{rear}} - P^*_{\text{front}}\right)$$





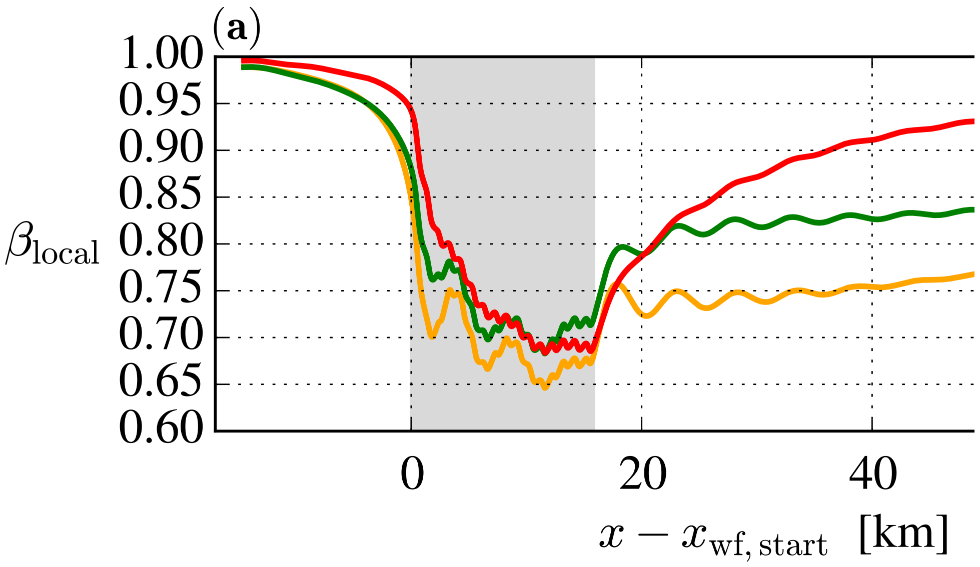
$\Delta M_{\mathrm{adv}} + \Delta M_{\mathrm{PGF}}$: AP approximation

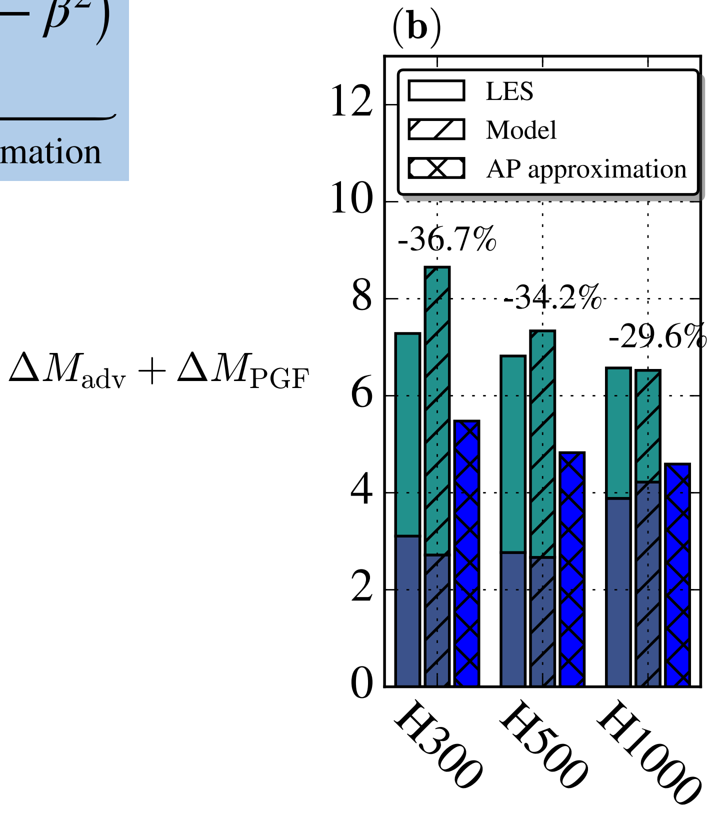
AP approximation:

$\beta_{\mathrm{local}}(0)^2 + \beta_{\mathrm{local}}(L)^2 \approx 1 + \beta^2$

Models and advection-pressure (AP) approximation:

$$\Delta M_{\mathrm{adv}} + \Delta M_{\mathrm{PGF}} = \underbrace{\frac{H_F}{LC_{f0}}\left(-\beta_{\mathrm{local}}(0)^2 - \beta_{\mathrm{local}}(L)^2 + 2\right)}_{\text{combined models}} \approx \underbrace{\frac{H_F}{LC_{f0}}\left(1-\beta^2\right)}_{\text{AP approximation}}$$





ΔM_{Cor}-model

Exact:

$$\Delta M_{\text{Cor}} = \frac{2}{C_{f0} Ro_{F0}} \left(\frac{[U_2] - [U_2]_0}{U_{F0}} \right)$$

Model:

$\Delta M_{\text{Cor}} = 0$

$$\Delta M_{\text{Cor}} \equiv \frac{X_{\text{Cor}} - X_{\text{Cor0}}}{X_{F0}}$$

$$X_{\text{Cor}} \equiv V_{\text{cv}} \rho f_c [U_2]$$

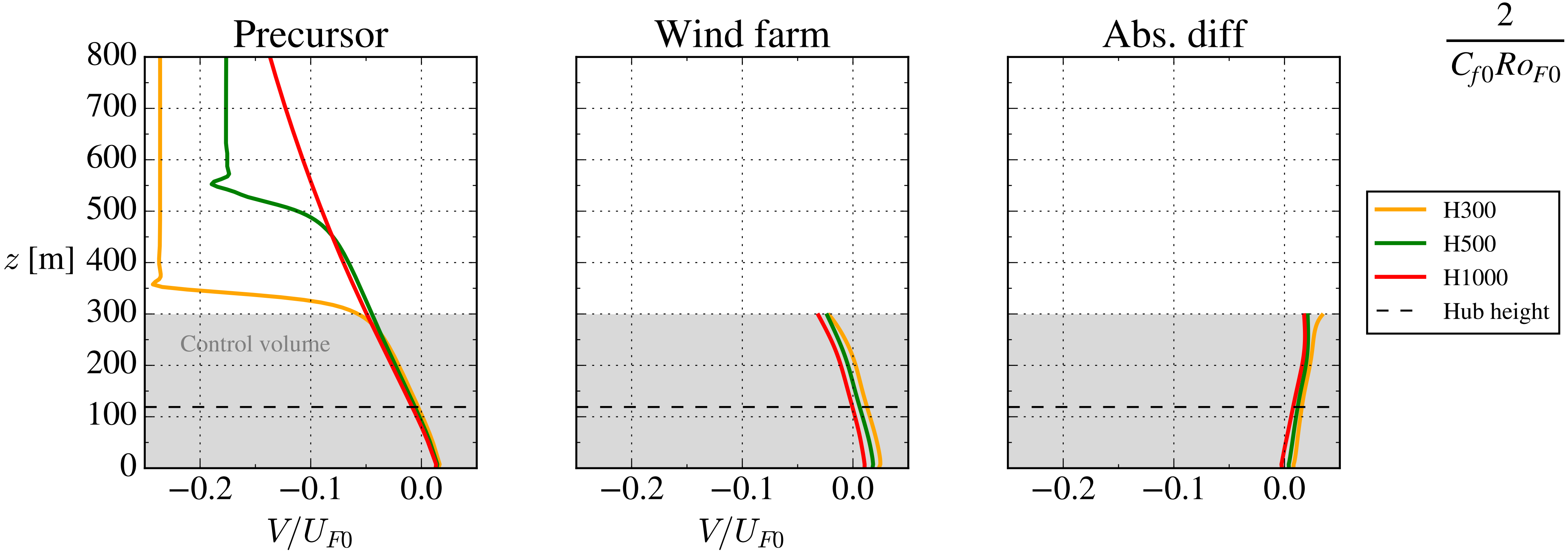
$$Ro_{F0} \equiv \frac{U_{F0}}{f_c H_F}$$

<u>For H{300,500,1000}:</u>

$Ro_{F0} \approx 280$

$C_{f0} \approx 0.0018$

$\frac{2}{C_{f0} Ro_{F0}} \approx 4$



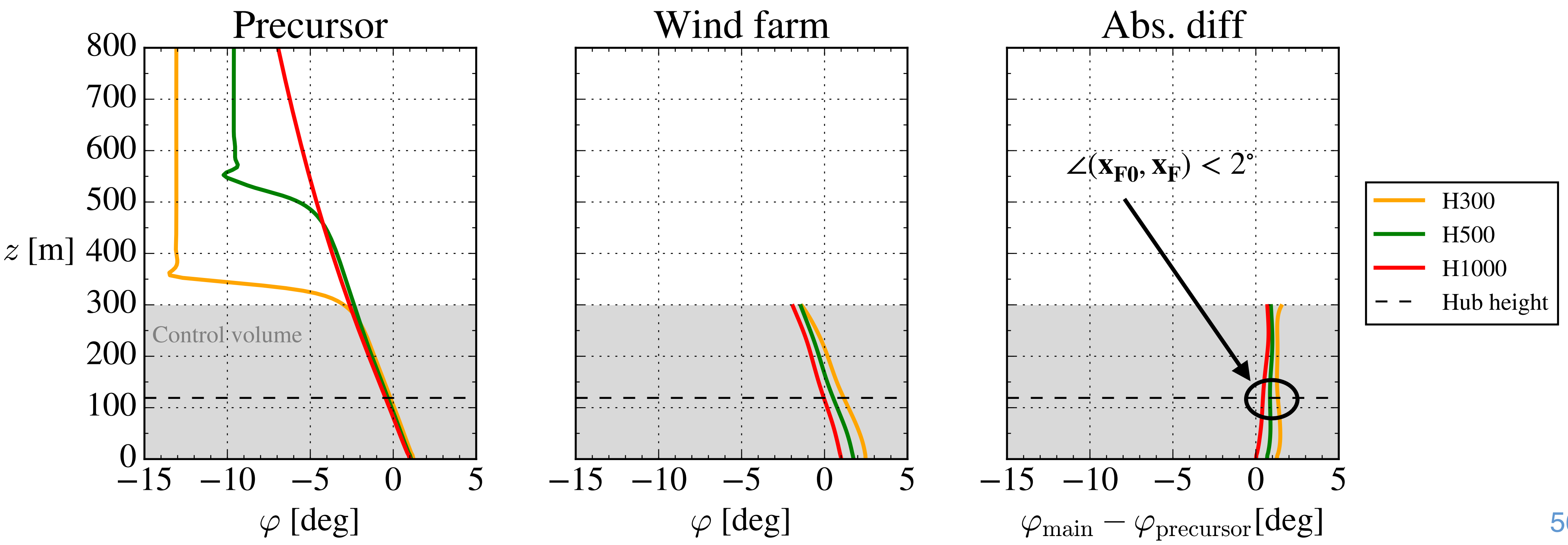
UNIVERSITY OF OXFORD

Nishino, Takafumi, and Thomas D. Dunstan. "Two-Scale Momentum Theory for Time-Dependent Modelling of Large Wind Farms." *Journal of Fluid Mechanics* 894 (2020): A2. https://doi.org/10.1017/jfm.2020.252.

Wind direction

In Nishino & Dunstan (2020), there is a discussion about the coordinate system.
They define:

- x_{F0}: direction of the flow at hub-height in the precursor.
- x_F: direction of the flow at hub-height in the main simulation.



ΔM_{turb}-model

Exact:

$$\Delta M_{\text{turb}} = \frac{\int_{\Omega_{\text{cv}}} \tau_{1j} dA_j - \int_{\Omega_{\text{top}}} \tau_{1j,0} dA_j}{X_{F0}}$$

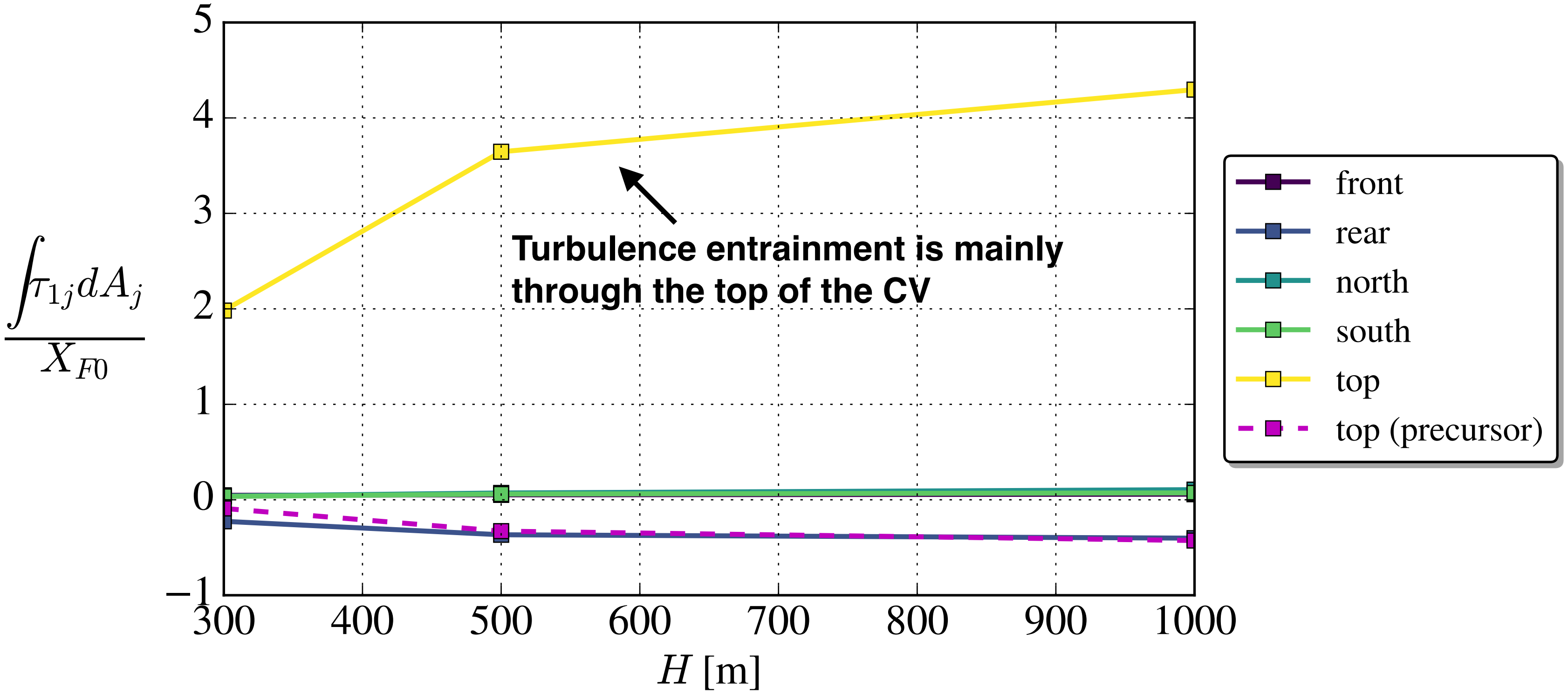
Model:

$$\Delta M_{\text{turb}} = \frac{\tau_t - \tau_{t0}}{\tau_{w0}}$$

$$\Delta M_{\text{turb}} \equiv \frac{X_{\text{turb}} - X_{\text{turb}0}}{X_{F0}}$$

$$X_{\text{turb}} \equiv \int_{\Omega_{\text{cv}}} \tau_{1j} dA_j$$

$$\tau_t \equiv \int_{\Omega_{\text{top}}} \tau_{13} dA$$



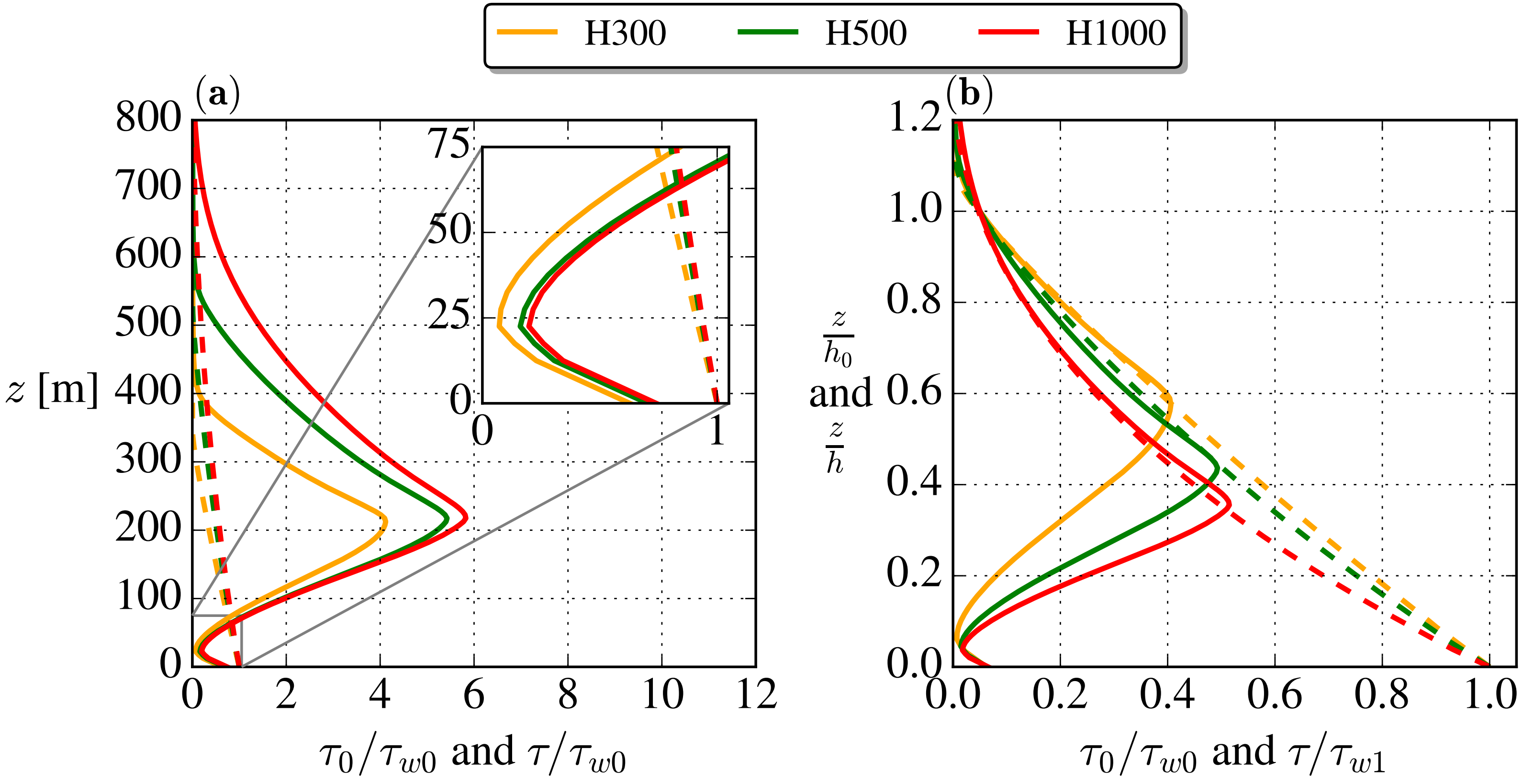
$\tau_{w1} \equiv M\tau_{w0}$

τ_t-model (1/2)

Self-similarity: $\dfrac{\tau_0\left(z/h_0\right)}{\tau_{w0}} = \dfrac{\tau(z/h)}{\tau_{w1}}$ (for $z > z_{\text{hub}} + D/2$) $\Rightarrow$

Model:

$$\tau_t = \tau_{w1}\frac{\tau_0\left(H_F\frac{h_0}{h}\right)}{\tau_{w0}}$$



$\tau_{w1} \equiv M\tau_{w0}$

τ_t-model (2/2)

Linear $\tau_0(z)$-profile: $\tau_0(z) = \tau_{w0} + \frac{\tau_{t0} - \tau_{w0}}{H_F} z \quad , \quad 0 < z < H_F$ $\Rightarrow$

Model:

$$\tau_0\left(H_F \frac{h_0}{h}\right) = \tau_{w0} + (\tau_{t0} - \tau_{w0})\frac{h_0}{h}$$

$\Downarrow$

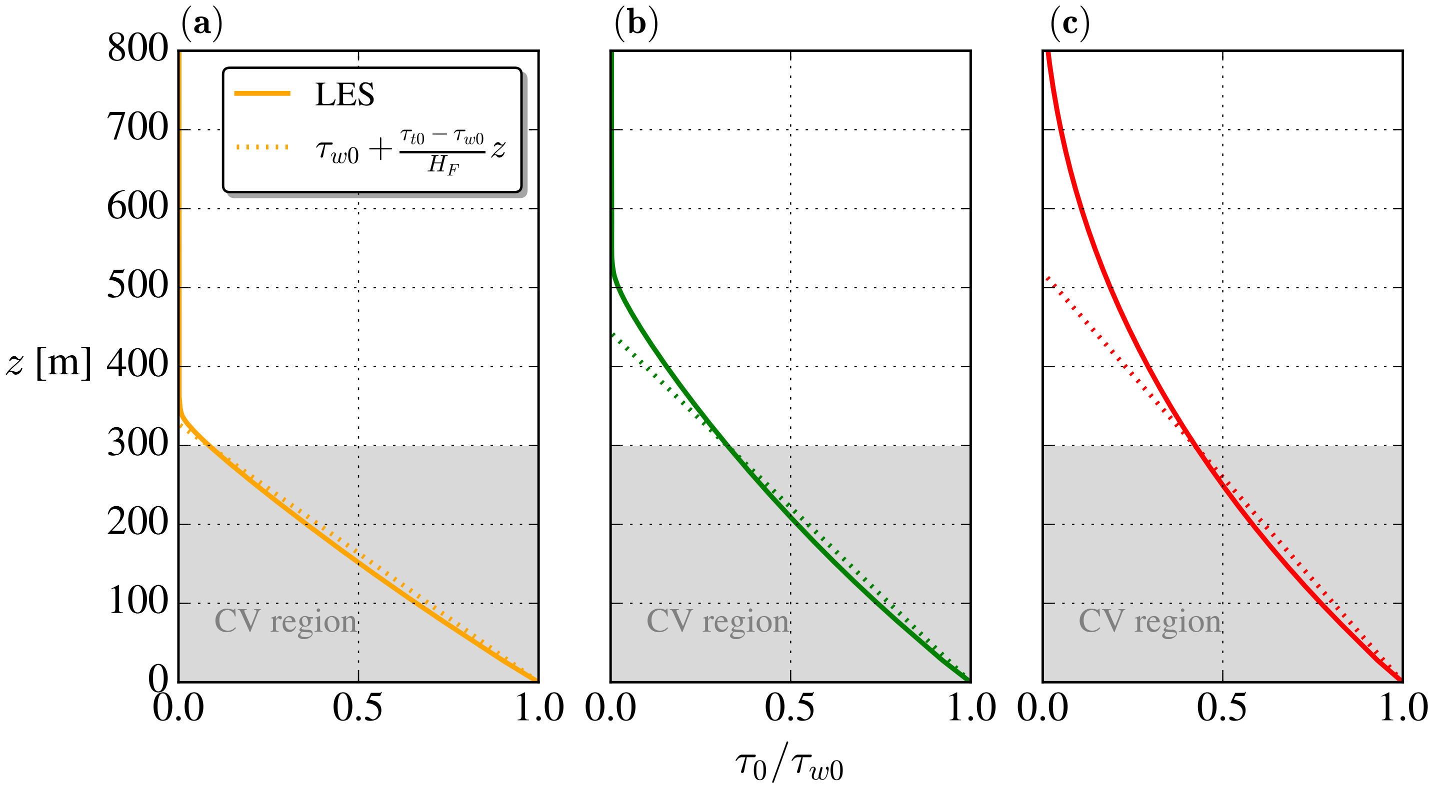
Model:

$$\tau_t = \frac{\tau_{w1}}{\tau_{w0}} \tau_0\left(H_F \frac{h_0}{h}\right) = M\left(\tau_{w0} + (\tau_{t0} - \tau_{w0})\frac{h_0}{h}\right)$$

$\Downarrow$

Model:

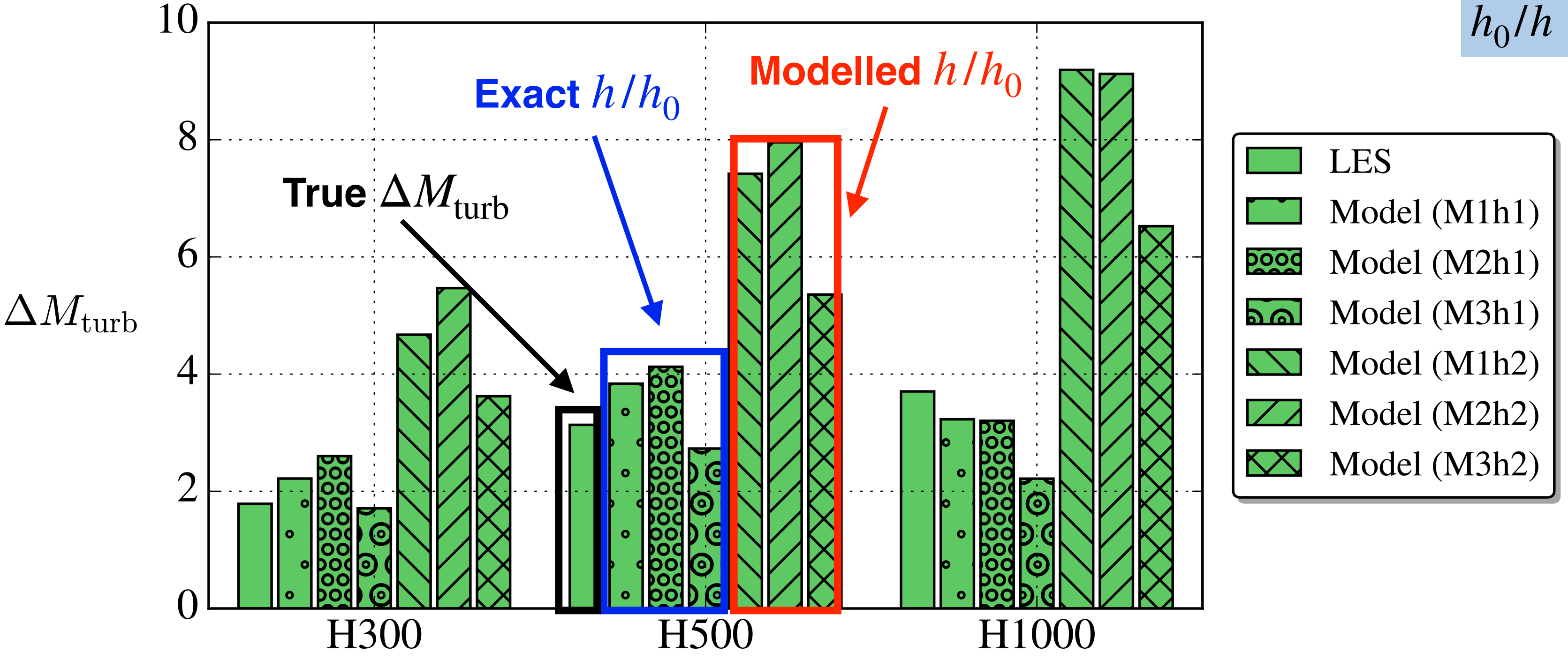
$$\Delta M_{\text{turb}} = \frac{\tau_t - \tau_{t0}}{\tau_{w0}} = M + M\left(\frac{\tau_{t0}}{\tau_{w0}} - 1\right)\frac{h_0}{h} - \frac{\tau_{t0}}{\tau_{w0}}$$



Test of ΔM_{turb}-model

More complex to test ΔM_{turb}, because it is an implicit expression.
It thus depends on:

- The other models: $\Delta M_{\text{adv}} + \Delta M_{\text{PGF}}$.
- The BL height ratio model: h/h_0



Model:

$$\Delta M_{\text{turb}} = M + M\left(\frac{\tau_{t0}}{\tau_{w0}} - 1\right)\frac{h_0}{h} - \frac{\tau_{t0}}{\tau_{w0}},$$

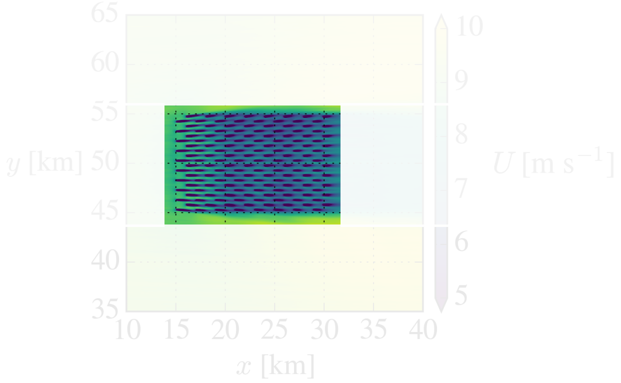
where

$$M = \frac{1 + \Delta M_{\text{adv}} + \Delta M_{\text{PGF}} - \frac{\tau_{t0}}{\tau_{w0}}}{\left(1 - \frac{\tau_{t0}}{\tau_{w0}}\right)\frac{h_0}{h}},$$

and

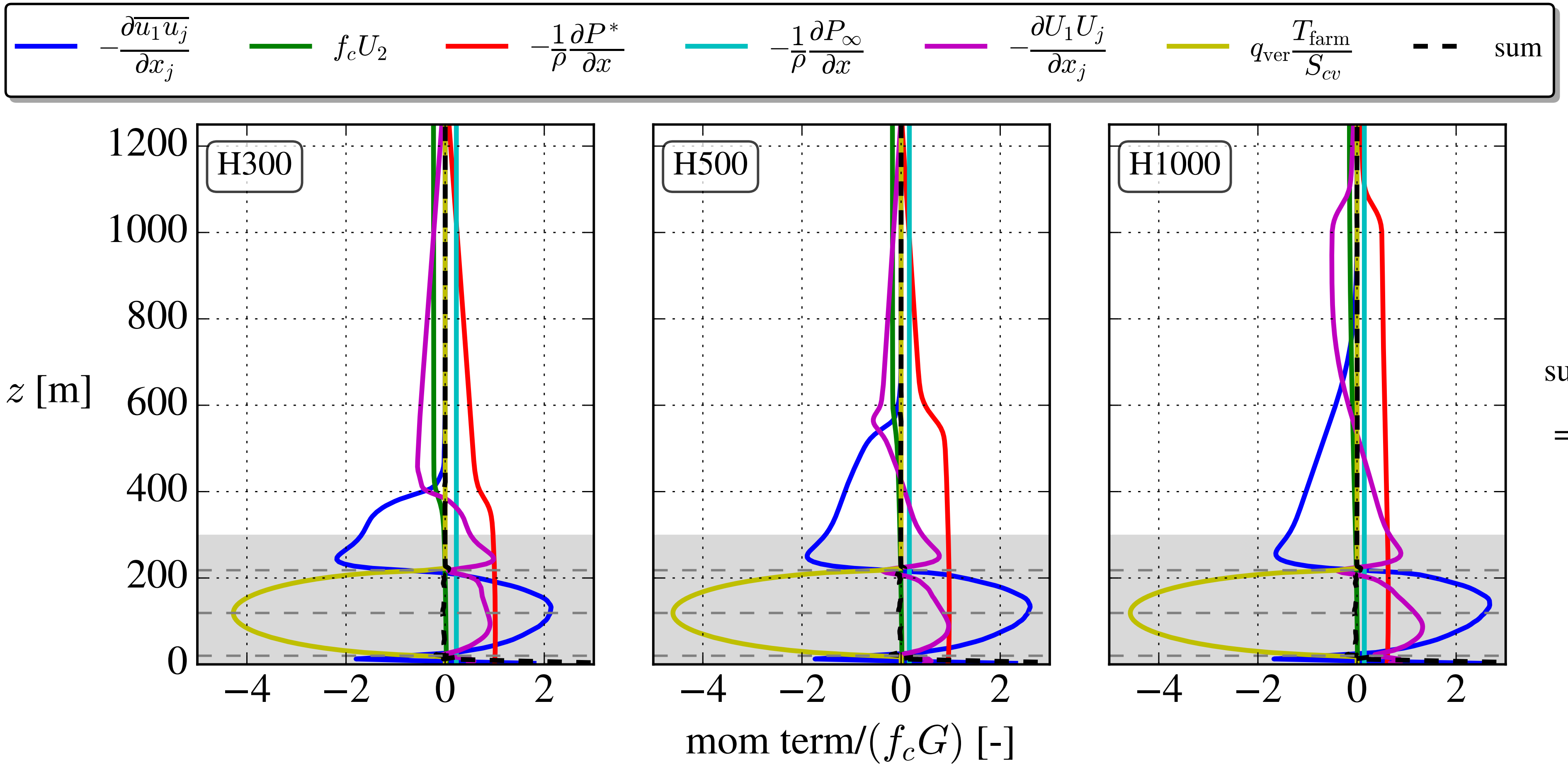
$$h_0/h = \beta\,.$$

Model:
$\Delta M_{\text{uns}} = 0$



ΔM_{uns}-model

Farm-averaged momentum balance



$$\text{sum} \approx 0 \Rightarrow \frac{\partial U}{\partial t} \approx 0$$
$$\Rightarrow \Delta M_{\text{uns}} \approx 0$$